RELIABILITY BASED DESIGN OF REINFORCED CONCRETE CIRCULAR SLABS

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By

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CERTIFICATE

This is to certify that the thesis entitled 'Reliability Based Design of Reinforced Concrete Circular Slabs' submitted by Mr. G.V.V.G. Vara Prasad in partial fulfilment of the requirements for the degree of Master of Technology of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out under my supervision and has not been submitted elsewhere for a degree.

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CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

A_{st}	=	Area of steel
b	=	Metre width of slab
C	=	Constant
C_c	=	Cost of ready mix concrete per unit volume
C_s	=	Cost of reinforcing steel per unit weight
C_f	=	Cost of thickness related formwork per unit area
d	=	Depth of slab
E_s	=	Youngs modulus of steel
$f(\vec{X})$	=	Multivariate function (objective function)
f_{ck}	=	Characteristic strength of concrte
f_y	=	Tensile stress in steel
$f_L(l)$	=	Density function for load
$f_R(r)$	=	Density function for resistance
$g(\vec{X})$	=	Constraints of $f(\vec{X})$
K	=	0.776
L	=	Load
$\bar{L}$	=	Mean load
L_o	=	Average load value
$\hat{L}$	=	Random variable with mean value $\bar{L}$
m	=	$\frac{(f_y/f_{ck})}{100}$
M_L	=	External moment
M_R	=	Moment of resistance

$\bar{M}_R$	=	Mean expected moment of resistance of slab per metre width
$\bar{M}_L$	=	Mean expected design moment of slab per metre width
N_r	=	Correction factor for resistance
N_l	=	Correction factor for load
P	=	Non-dimensionalised uniformly distributed load
p	=	Percentage area of steel
p_o	=	Percentage area of balanced steel reinforcement
p^*	=	Uniformly distributed load on the slab
p_f	=	Probability of failure
Q_R	=	Nondimensionalised moment
R	=	Radius of slab
R(Z)	=	Reliability
r_k	=	Penalty parameter
$\bar{R}$	=	Mean value of resistance
$\hat{R}$	=	Random variable with mean value $\bar{R}$
SF	=	Safety factor
$\bar{S}_R$	=	Standard deviation of resistance
$\bar{S}_L$	=	Standard deviation of load
S_y	=	Standard deviation of Y
S_{M_R}	=	Standard deviation of $\bar{M}_R$
S_{M_L}	=	Standard deviation of $\bar{M}_L$
S_{Q_R}	=	Standard deviation of Q_R

$V_{A_{st}}$, V_{f_y}, $V_{f_{ck}}$, V_d, V_b, V_R, V_{p*}, V_{E_s} are coefficients of variation of respective suffixes

w_L	=	Live load
X_i	=	ith design variable
X_u	=	Neutral axis depth
$\vec{X}$	=	n - dimensioned vector
Y	=	Difference between resistance and load
Z	=	Reliability parameter
δ_R	=	Coefficient of variation of R
δ_L	=	Coefficient of variation of L
Δ_R	=	Standard deviation of error in $\hat{R}$
Δ_L	=	Standard deviation of error in $\hat{L}$
$\phi(\vec{X},r_k)$	=	Penalty function.

ABSTRACT

It is well known that conventional design practice of RCC members often results in overly conservative designs. It is also known that the degree of conservatism can not readily be estimated by current methods. The statistical nature of design variables is usually ignored in conventional practice. The growing need for answers to such questions - How safe is design ? How can designs be made to specified levels of adequacy (reliability)? - leads directly to probabilistic methods. The present work is a step towards this direction.

This work can be broadly classified into two parts.

(i) Reliability based design of RCC circular slabs with fixed as well as simply supported edge conditions.

(ii) Optimum cost design of RCC circular slabs with a reliability constraint.

The first part of the work consists of the parametric study of the reliability based design of under-reinforced circular slabs with uniformly distributed load. By changing the value of each parameter turn by turn the effect of design variables on the area of steel reinforcement is studied. The design parameters included in parametric study are depth

of the slab, characteristic compressive strength of concrete, characteristic yield strength of steel, reliability parameter and coefficient of variation of the design variables. It is found that the area of steel is very much sensitive to the yield strength of steel and reliability paramter. The steel area is also sensible to the coefficient of variation. By grouping the parameters in the nondimensional form, reliability analysis is carried out and design curves are presented as per Indian Standard Code Practice (IS 456-1978). Two examples are taken for illustration of the use of the design charts.

Finally the optimum cost design of under reinforced concrete RCC circular slabs is studied with a reliability constraint. The design is considered safe and adequate if the expected moment capacity is greater than design moment capacity by a certain number of standard deviations. The effect of coefficient of variation and change in cost ratio on the optimum design cost is also studied. The optimal cost design problem is cast as a nonlinear mathematical programming problem and is solved by using sequential unconstrained minimization technique.

CHAPTER 1

INTRODUCTION

1.1 General:

The fundamental role of probability theory in safety and performance analysis is widely recognized in all branches of engineering. Despite more than 20 years of recognition in structural engineering literature , the probabilistic methods have not yet been explicitly adopted as a basis for any standard code of practice. The probability theory provides a more accurate engineering representation of reality. Many leading civil engineers in several countries have written [1] about the statistical nature of loads and material properties. It has been demonstrated that such uncertainty in applied forces and in structural resistance implies uncertainty in structural performance and that this uncertainty can be analysed rationally only with probability theory.

The conclusion is that, if structural safety is to be placed in a position where it can be discussed quantitatively, it must be treated probabilistically. Several authors suggest or imply that the traditional nominal strengths, nominal loads and safety factors (or load factors) be replaced by an allowable 'probability of failure' calculated on the basis of observed frequency distribution of strengths and loads. In this way

it is claimed that more realistic statistical models would produce better engineering results.

'Reliability' is the capacity of a structural system to perform its assigned functions under specified environmental conditions for a specified period of time. In otherwords, it is a measure of the adequacy of a design in its intended environment, whenever physical or other factors that must be considered are essentially probabilistic.

1.2 Reasons for Slow Inclusion in Codes of Practice:

Although a considerable amount of research effort has been devoted to the study of probabilistic approach of design, the progress towards its implementation in standard codes of practice is rather slow due to the following reasons:

(i) For numerous reasons standard codes must remain uncomplicated in form and must have a basis that is straight forward to understand. The reliability analysis calculations in the literature[1] do not promise either benefit. Although a new code format that the profession believed to have a more rational basis and to lead to more consistent safety could presumably afford some minor increase in complexity over present codes.

(ii) The assessment and justification of a numerical value for the 'acceptable risk' of failure.

(iii) Engineers recognize that present safety factors account for many uncertainities which are seldom included in the observed statistical data. These other factors include those about which there is incomplete knowledge and those for which the engineering analysis is possible but not economically justified (e.g. the relationship between laboratary strength and field strength ; local ,long term and non-linear stress distributions; actual versus assumed uniform load distribution, perhaps even minor engineering miscalculations and construction blunders whose guranteed elimination would require many hours of technical man power). In short, much of the uncertainty in structural performance predictions is of a fundamentally non statistical nature. This is not to say however, that it is nonprobabilistic in nature.

(iv) On a more esoteric level, it is unreasonable to believe that a complete, strictly statistical probabilistic analysis of structural safety will even be feasible for standard structures. The available data will always be too limited to provide wholly reliable estimation of failure probabilities of the order of magnitude necessary for adequate public safety. The sample sizes required are in multimillions. Extrapolation of mathematical models of frequency distributions will always be a part of these analyses.

There are, then sound reasons why the suggested adoption of probabilistic safety analysis has not been implemented in

standard structural engineering practice. Nonetheless, the fundamental arguments for a statistical basis remain. It still promises to provide a more rational, quantitative representation of engineering design.

1.3 Fallacies in Designing by Safety Factors:

The quantity in use to maintain a proper degree of safety in a conventional design is usually referred to as the factor of safety. The following are the several other definitions in its broader sense, i.e. ratio of the maximum safe load to the normal service load, ratio of mean strength to mean load or mean failure governing strength to mean failure governing stress.

There are several important, relatively new circumstances which have brought the inadequacy of the factor of safety concept to the fore. Some of them may be mentioned as:

(1) The statistical nature of design variables is usually ignored in conventional practice. The conventional approach in design practice may be compared to a kind of worst-case analysis. The maxima of loading and the minima of strength are treated not only as representative of design situations, but also of simultaneous occurrence. This is the basis on which unknown parameters are computed. Actually, magnitude and frequency relationships for both load and strength must be considered to avoid unrealistic results. If an extremely

large load must act on an extremely low value of strength to induce a failure, then the probability of such simultaneous occurrences is very important. Fig. 1.1 picture the type of comparison discussed above.

(2) The safety factor concept completely ignores the facts of variability that result in different reliabilities for the same safety factor. The Fig. 1.2 indicates a finite probability of failure whose magnitude is a function of the degree of overlap of the two distributions. As the overlap increases, the shaded area and consequently failure probability increase proportionately. Thus there is a rational explanation for changes in failure probability which accompany changes in the load-strength distribution overlap.

Three possibilities exist in which a safety factor (SF) may be maintained as failure probability varies:

(i) Mean load and mean strength may be changed in the same proportion with no change in the standard deviations. Thus,

$$SF = \frac{R}{L} = \frac{K_1 R}{K_1 L} = K \quad \ldots \quad (1.1)$$

where K_1R and K_1L reflect a shift in mean strength and load respectively. The Fig. 1.3(a) illustrates the shift for $K_1 > 1$. This demonstrates the limitation of safety factor as an indicator of failure incidence and of design safety.

(ii) Failure probability varies with the mean values and distributions are held constant and the standard deviations are varied as shown in Fig. 1.3(b). Since the mean values are not changed, the safety factor remains the same, but failure probability does change.

(iii) It is possible to change both mean and standard deviation values without changing the safety factor.

The Table I clearly indicates that even though the factor of safety remains same, the probability of failure may vary significantly.

TABLE I

SAFETY FACTOR AND RELIABILITY

Mean Value		Standard Deviation		Safety Factor	Reliability
Strength $\bar{R}$(MPa)	Load $\bar{L}$(MPa)	Strength S_R(MPa)	Load S_L(MPa)		
250	100	10	15	2.5	0.9166
250	100	80	30	2.5	0.9099
250	100	100	60	2.5	0.8997
250	100	250	250	2.5	0.6628

Bar (–) over the variable indicates their mean value.

(3) The day to day increasing complexities and environmental actions presenting a much broader spectrum of loading (for 3-d structures, concrete structures, marine structures, nuclear weapons and space explorations etc.) necessiate the study of statistics of extremes for the rational design of such structures.

1.4 Need for Probabilistic Approach to Reinforced Concrete Members:

The actual strength of a reinforced concrete member (RCC) differs from the nominal strength calculated by the design engineer due to variations in the material strengths and geometry of the member, as well as the variabilities inherent in the equations used to compute the member strength. Similarly, designers use constant nominal values of loads in their calculation of forces, but the actual loads are variable. This variability in strength and loading is accounted for in one form or another in safety provisions of all existing codes. As a first step in the calculation of resistance factors, it is necessary to study the variability of the strength of reinforced concrete in flexure, shear, combined axial force and flexure etc. This requires a knowledge of the variability of the parameters that effect the strength of RCC members. These are the concrete strength in compression and tension, the yield strength of steel and

the dimensions of the cross section of the member. In practice the compressive strength of concrete varies with in the same mix also. For considering this variability in compressive strength of concrete IS code of practice[32] is specifying characteristic strength (i.e. more than 5 percent of the test results should not fall below the specified strength). Even at the time of construction also the cross sections of the members may be slightly different from section to section because of the construction faults.

In reliability approach all these variabilities are taken in to account. Because of these variabilities in materials and material strengths of RCC members, it is more appropriate to go for a reliability approach.

1.5 Steps Involved in Probabilistic Analysis and Design:

The various steps involved in probabilistic analysis and design may be presented in a flow chart as shown in Fig. 1.4.

1.6 Scope of the Thesis:

The major interest of the thesis is to study the design aspects of RCC circular slabs with preassigned probability of failure. The work can be broadly classified into three parts.

i) Parametric study of the design formulation for RCC circular slabs, finding the area of steel by changing a particular design variable while all other variables are kept constant.

ii) Presentation of some design charts for RCC circular slabs with reliability parameter as per Indian Standard Code of Practice for Plain and Reinforced Concrete(32).

iii) Finding the minimum cost of RCC slabs with reliability as a constraint.

In the first two chapters, the general aspects of reliability based designs are reviewed, fallacies in safety factor concepts discussed and literature is reviewed.

The Chapter 3 contains design formulation for RCC circular slab design and some readymade design charts for free edge conditions as well as for fixed edge conditions.

The Chapter 4 deals with some studies on optimum cost of the RCC slabs with reliability as a constraint.

Some design examples, discussion and conclusions are presented in Chapter 5.

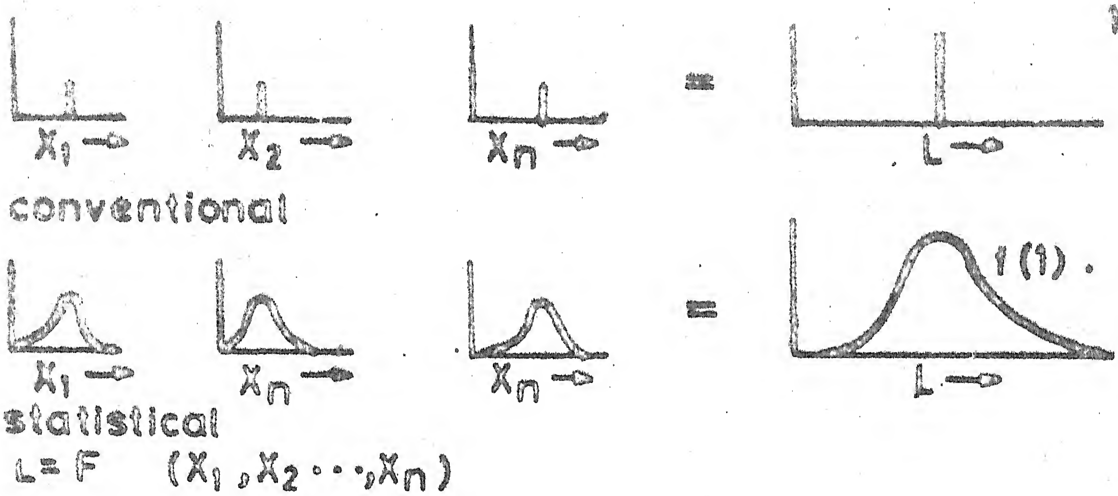


FIG.1.1 PARAMETER REPRESENTATION

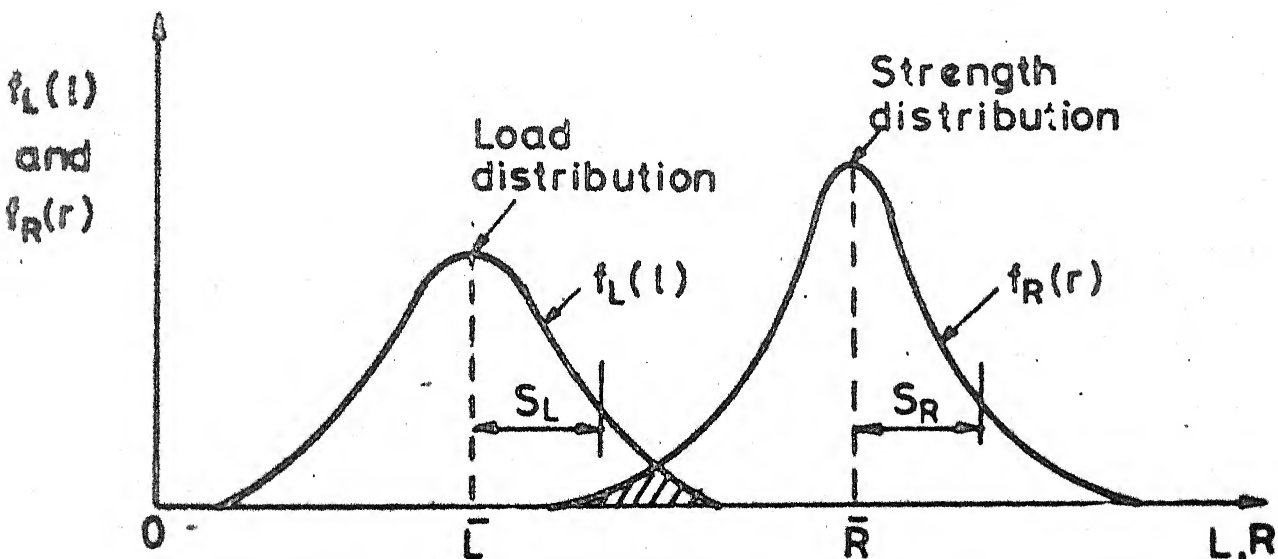


FIG.1.2 FREQUENCY DISTRIBUTION OF THE LOAD (L) AND MATERIAL RESISTANCE (R) INCLUDING PROBABILITY OF FAILURE (Shaded area)

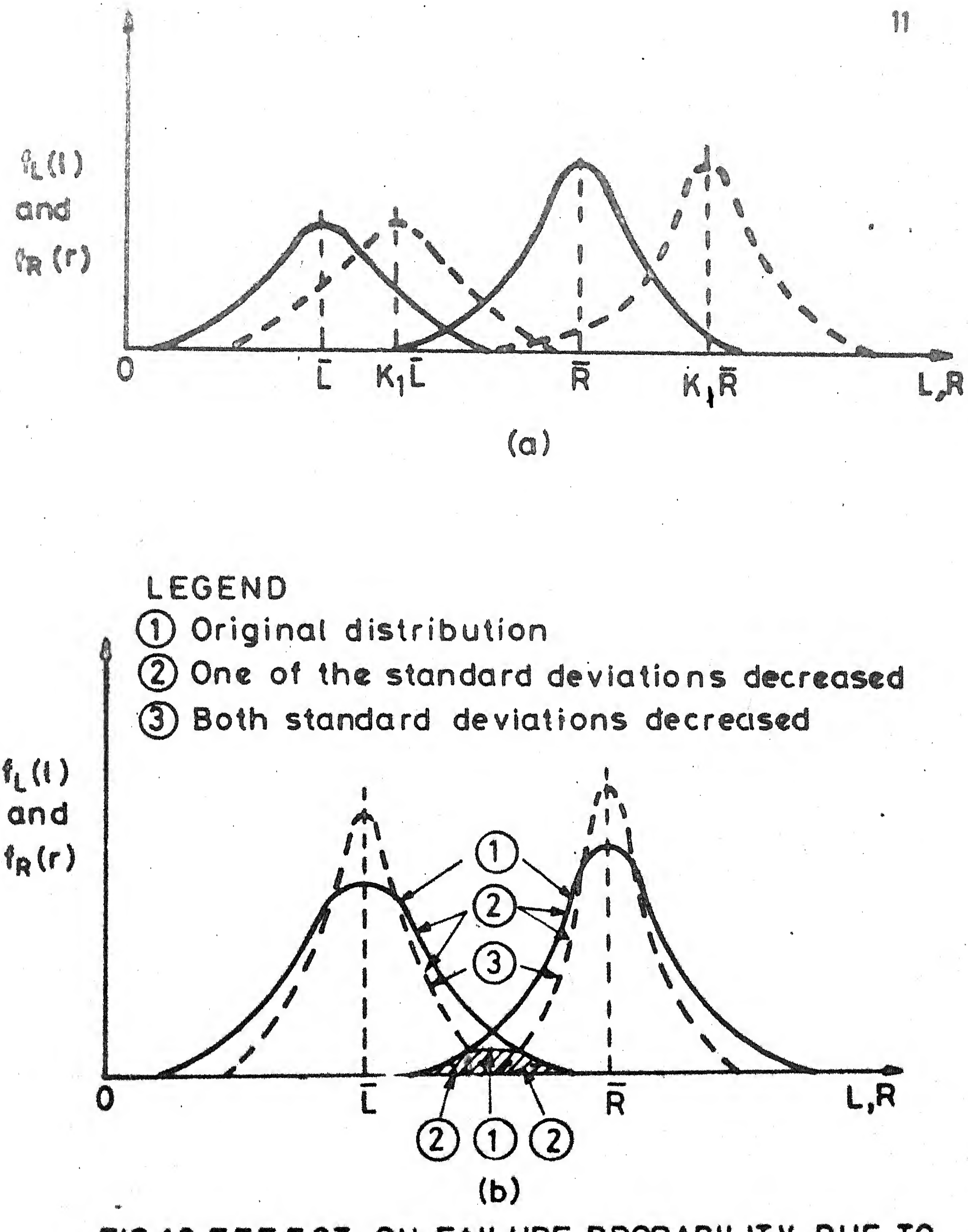


FIG. 1.3 EFFECT ON FAILURE PROBABILITY DUE TO CHANGE IN MEAN AND STANDARD DEVIATION VALUES

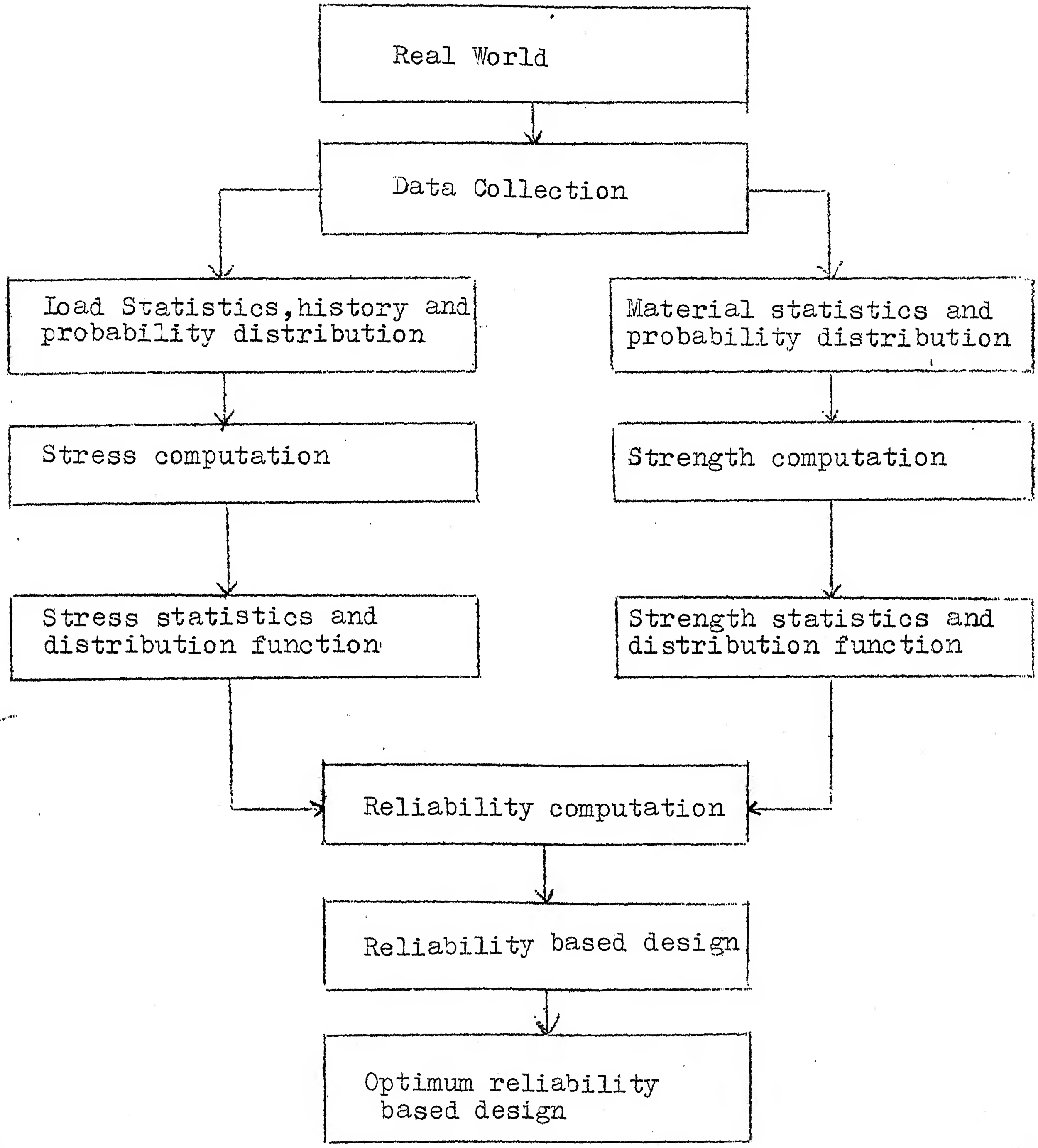


Fig. 1.4 : Steps Involved in Probabilistic Analysis and Design

CHAPTER 2

LITERATURE REVIEW AND RELIABILITY THEORIES

2.1 Literature Review:

Only in recent years a sizeable quantity of literature has appeared in the area of reliability based design and probability of failure of structures due to random variables of loads and strengths.

Literature on probabilistic distributions for strength of concrete beams, safety of structures, risk analysis, probabilistic design of members and optimum designs with reliability constraints is reviewed.

Among the first to recognize the concepts of an associated probability of failure in any design are Freudenthal(1), Pugsley(2) and Prot(3). Freudenthal analyzed the safety factor in engineering structures while Pugsley pleaded for the assessment of probability of failure and the collection of statistical data of loads and structural properties.

Freudenthal(1) analyzed the safety factor in engineering structures in order to establish its magnitude. He derived the safety factor from absolute and measurable physical properties and phenomenon. External conditions and pertinent physical properties and phenomenon were analysed under the common aspect of their relation to the magnitude

of safety factor. Pugsley[2] discussed the method of design incorporating the safety assumptions as well as the efficiency concepts. The terms like structural efficiency and structural safety were defined and a strut problem was considered for the classification of concepts.

Freudenthal[4] outlined the procedure for the numerical evaluation of factor of safety based upon the concept of unserviceability or probability of failure.

Freudenthal et.al.[5] correlated the factor of safety with probability of survival or failure. It also provides guidance by suggesting and illustrating techniques that may be of considerable value in studying certain phases of the safety (reliability) problem.

Benjamin[6] discussed the superiority of probabilistic designs over conventional deterministic procedures in respect of informational content, modeling of reality, refinement of analysis and design, decision making and the resulting structures.

Sexsmith[7] presented procedures for determining the probability distribution of the safety margin and related quantities for several types of reinforced concrete members. Data were extracted from previously published test results, and were separately analyzed for each of three common

structural types; beams without compression reinforcement, beams with compression reinforcement and tied columns.

Benjamin and Lind[8] discussed one possible application of probabilistic procedures to back-up a deterministic code. Subjective probability measures were used to obtain model parameters and then some applications of the results were presented.

Shah[9] outlined reasons for interest in probabilistic studies of safety principal considerations in formulating probabilistic codes were discussed and a simple strength modelling example was introduced. The probability implications of ACI 318-63 ultimate strength design procedures were presented.

Cornell[10] introduced a frame work for structural code in which probability was used to enhance realism. He treated all the uncertainities through standard deviation and presented a format of ACI code of practice (ACI 318-63).

Allen[11] has summarized the results of a probabilistic study of ultimate moment and ductility ratio of reinforced concrete. The results presented are useful in assessing existing design formulas and safety provisions.

Moses and Stevenson[12] presented methods for incorporating reliability analysis into optimum design procedures. Examples were given for multimember elastic

truss design and frames designed according to limit design theory. The approach adopted was to design for a specified probability of failure, in which the failure probability was evaluated from a sequence of numerical integrations. Ang[13] has summarized some basic probabilistic models and he had shown that practical formulation of risk based design and evaluation of safety and performance, including repeated loading and fatigue can be developed consistently within the existing probabilistic models of structural reliability. Chandrasekhar[14] has analyzed number of sets of field data and found that the probability of failure of concrete cube is about 0.12 for the strength ranging from 35 MPa and the individual variation of strength from the mean value is ± 10 percent.

Ang[15] identified the requirements and conditions necessary for implementing the probabilistic approach and has shown that the explicit consideration of risk in the evaluation of performance and formulation of design are now practically feasible and desirable.

Ranganathan and Dayaratnam[17] used Monte Carlo Simulation Technique to evaluate the probability of failure of a prestressed concrete flanged section. The failure of the section at limit state of strength was divided into two cases: (i) under-reinforced and (ii) over-reinforced and

the probability of failure was evaluated based on the above two conditional failure events.

Shah and Sexsmith [18] described the general philosophy of developing probability based codes. Various probability based improvements were suggested for the current ACI Code of practice.

Mirza et.al.[2] reviewed the variability of strength and stiffness normal -weight structural concrete and representative distributions were suggested for use in estimating the effect of these variations on the strength of reinforced concrete elements.

Ellingwood[21] described the results of a study where objective was to assist in the development of probabilistic limit state design criteria for use in the United States of America.

Mirza and MacGreger[22] documented the assumptions and analysis made in the derivation of resistance factors for reinforced concrete design compatible with the load factors in the National Building Code of Canada. Statistical estimates were made of the variability of the strength of reinforced concrete members based on the available data on the variability of concrete and steel dimensions.

MacGregor et.al. [23] presented the results of studies of the variability in the strengths of reinforced concrete

beams and columns. They also presented the coefficients of variation of depth and steel reinforcement of slabs. For variabilities of member strengths, they had interpreted the results of Monte Carlo Simulations; these inturn were based on statistical distributions obtained from literature.

Roy[24] presented some design charts for reliability based design of RCC beams. He has taken the coefficients of variation for different design variables from test results compiled by Allen on more than 100 beams. Finally optimization of beams with reliability constraint was presented.

Moses and Kinser[25] presented optimum sizing of elements for a multielement, multiload conditions structure with safety defined interms of allowable probability of failure. They presented weight minimization and probability analysis which are applicable to any elastically designed structure and can treat any frequency distribution for each loading and element strength.

Rao[26] discussed the optimum cost design of under-reinforced concrete beams by treating all the design parameters as random variables and considering the reliability as the constraint. Numerical examples were presented to illustrate the effectiveness of the method.

2.2 General Expression for Reliability:

For deriving a general expression for reliability, applied load and resistance of the structure (or system) of unspecified distribution are considered.

The reliability of a structural system is determined from the basic concept that a no failure probability exists when resistance (R) of the component or member is not exceeded by the applied load (L) or resistance > load applied.

Let the density function for load (L) be denoted by $f_L(l)$ and that for resistance (R) by $f_R(r)$. Then by definition,

$$\text{Reliability} = P\ (R > L) = P\ (R - L > 0) \quad \ldots \quad (2.2.1)$$

The shaded portion in the Fig. 1.2 shows the interference area, which is indicative of the probability of failure. Figure 2.1 shows the enlarged portion of the Fig. 1.2 . The probability of a load value L_o lying in a small interval of width dL is equal to the area of the element dL, i.e.

$$P\ \left(L_o - \frac{dL}{2} \leq L \leq L_o + \frac{dL}{2} \right) = f_L(L_o)\ dL \ldots \quad (2.2.2)$$

The probability that the strength R is greater than a certain load L_o is given by

$$P\ (R > L_o\ ) = \int_{L_o}^{\infty} f_R(r).dR \quad \ldots \quad (2.2.3)$$

The probability for the load value lying in the small interval dL and the strength R exceeding the load given by the small

interval dL under the assumption that the load and strength random variables are independent is given by

$$f_L(L_o)dL \int_{L_o}^{\infty} f_R(r)\ dR \quad \ldots \qquad (2.2.4)$$

Now the reliability of the element is the probability that the strength R is greater than the load L for all possible values of the load L and hence is given by

$$\text{Reliability} = \int_{-\infty}^{\infty} f_L(l) \left(\int_{L}^{\infty} f_R(r).\ dR \right) dL \quad \ldots \qquad (2.2.5)$$

Reliability can also be computed on the basis that the load remains less than the strength. The probability that the strength is with in a small interval dR is

$$P(R_o - \frac{dR}{2} \leq R \leq R_o + \frac{dR}{2}) = f_R(R_o)\ dR \ldots \qquad (2.2.6)$$

and the probability of the load being less than R_o is given by

$$P\ (\ L \leq R_o\ ) = \int_{-\infty}^{R_o} f_L(l).\ dL \quad \ldots \qquad (2.2.7)$$

Hence the reliability of the element (or member) for all the possible values of the strength R is

$$\text{Reliability} = \int_{-\infty}^{\infty} f_R(r) \left(\int_{-\infty}^{R} f_L(l) dL \right) dR \quad \ldots \qquad (2.2.8)$$

The unreliability or probability of failure, is defined as

Probability of failure = 1 - Reliability

= P (R ≤ L)

Thus

Probability of failure + Reliability = 1.

2.3 Reliability Computations for Normally Distributed Strength and Load:

Many random variables encountered in the physical sciences appear to be normally distributed and the normal distribution gives an adequate approximation to the distribution of many other measurable random variables.

Thus, if a complete theory of statistical inference is developed based on the normal distribution alone, then one has in reality a system which may be employed quite generally, because other distributions can be transformed to or approximated by the normal form.

If L and R are normal variables, then the density functions are

$$f_L(l) = \frac{1}{S_L\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{L-\bar{L}}{S_L}\right)^2\right] \quad -\infty < L < \infty \quad \ldots \quad (2.3.1)$$

$$f_R(r) = \frac{1}{S_R\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{R-\bar{R}}{S_R}\right)^2\right] \quad -\infty < R < \infty \quad \ldots \quad (2.3.2)$$

Reliability is the probability that strength exceeds the load

$R - L > 0$

or $Y > 0$

where $Y = R - L$

Then it is well known that the random variable Y is normally distributed with a mean of

$$\bar{Y} = \bar{R} - \bar{L} \quad \ldots \quad (2.3.3)$$

and standard deviation of

$$S_Y = \sqrt{S_L^2 + S_R^2} \quad \ldots \quad (2.3.4)$$

and the density function is given by Fig. 2.2.

where

$\bar{R}$ and $\bar{L}$ are the mean values of strength and load respectively.

S_R and S_L are the standard deviations of strength and load respectively.

The reliability can be expressed interms of Y as

$$\text{Reliability} = P\,(Y > 0)$$
$$= \int_0^\infty \frac{1}{S_Y\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{Y-\bar{Y}}{S_Y}\right)^2\right] dY \quad \ldots \quad (2.3.5)$$

If we let $Z = \left(\dfrac{Y-\bar{Y}}{S_Y}\right)$ then $S_y dZ = dY$.

When $Y = 0$, the lower limit of Z is given by

$$Z = \frac{0-\bar{Y}}{S_Y} = -\frac{\bar{R}-\bar{L}}{\sqrt{S_R^2 + S_L^2}} \quad \ldots \quad (2.3.6)$$

and when $Y \to +\infty$, the upper limit of $Z \to +\infty$.

Therefore

$$\text{Reliability} = \frac{1}{\sqrt{2\pi}} \int_{-\frac{\bar{R}-\bar{L}}{\sqrt{S_R^2+S_L^2}}}^{\infty} e^{-Z^2/2}\, dZ \quad \ldots \quad (2.3.7)$$

Clearly, the random variable $Z = (Y-\bar{Y})/S_Y$ (2.3.8)

is the standard normal variable.

Hence reliability can be found by merely referring to the normal tables (Appendix A).

Equation (2.3.6) which is used to find the lower limit of the standard normal variate Z, is commonly known as coupling equation.

Equation (2.3.7) can be rewritten as

$$\text{Reliability} = -\phi\left(\frac{\bar{R}-\bar{L}}{(S_R^2+S_L^2)^{1/2}}\right)$$

$$= 1 - \phi(Z) \qquad \ldots \qquad (2.3.9)$$

Where $\phi(Z)$ is the standardized cumulative distribution function.

2.4 Extended Reliability Theory:

If R and L are random variables, there is a finite probability of failure as defined earlier

$$P_f = P\ (R \leq L) \qquad \ldots \qquad (2.4.1)$$

In practice R and L are of course unknown, it is possible only to make predictions or estimates of what these are, or might be, using theoretical models; in particular, these predictions would normally be limited to the estimations of the means and variances of the relavent design variables.

Such theoretical predictions are invariably imperfect and thus will contain errors.

Assuming R and L as random variables, the uncertainities associated with the inherent randomness in R and L are modeled in equation (2.4.1), however, since there are also uncertanities associated with the errors of prediction, this is not sufficient. To be sure, the prediction errors must be included in the risk model; for this purpose, the extended reliability model is apropos, the essence of which can be described as follows.

2.4.1 Formulation of Extended Reliability Model:

Let $\hat{R}$ and $\hat{L}$ be the theoritical models of R and L, respectively; in order to compensate for any shortcomings in these models, apply correction factors N_r and N_l, such that

$$R = N_r \hat{R}$$
$$L = N_l \hat{L} \qquad \ldots \qquad (2.4.2)$$

Thus the probability of failure becomes

$$P_f = P\,(R \leq L)$$
$$= P\,(N_r\hat{R} \leq N_l\hat{L}\,) \qquad \ldots \qquad (2.4.3)$$

$\hat{R}$ and $\hat{L}$ are generally random variables with mean values $\bar{R}$ and $\bar{L}$, and standard deviations $S_{\hat{R}}$ and $S_{\hat{L}}$; the coefficients of variation $\delta_{P} = \dfrac{S\hat{R}}{\bar{R}}$; $\delta_L = \dfrac{S\hat{L}}{\bar{L}}$.

N_r and N_l should also be assumed to be random variables with mean values 1.0 and coefficients of variation Δ_R and Δ_L, representing the errors in $\hat{R}$ and $\hat{L}$ respectively. On the basis of first order theory, the mean values of R and L by virtue of equation (2.4.2) are

$$S_R \simeq \bar{N}_r \bar{R} = \bar{R}$$

and $$S_L \simeq \bar{N}_l \bar{L} = \bar{L}$$

in other words, the predicted means $\bar{R}$ and $\bar{L}$ are unbiased estimaters of the true means S_R and S_L, where as the total uncertainities associated with the prediction of and randomness of R and L are

$$V_R^2 \simeq \delta_R^2 + \Delta_R^2$$

$$V_L^2 \simeq \delta_L^2 + \Delta_L^2 \quad \text{respectively.}$$

The equation (2.4.3) can be written as

$$P_f = P(\hat{R} \leq N \hat{L}) \quad \ldots \qquad (2.4.4)$$

where $N = N_l/N_r$.

$\hat{R}$ and $\hat{L}$ may be assumed to be independent lognormal variates with means $\bar{R}$ and $\bar{L}$ and coefficients of variation δ_R and δ_L, whereas N_r and N_l are also lognormal with means 1.0 and coefficients of variation Δ_R and Δ_L; then the failure probability of equation (2.4.3) is given conveniently by

$$P_f \simeq 1 - \phi\left(\frac{\ln \bar{R}/\bar{L}}{\sqrt{V_R^2 + V_L^2}}\right) \quad \ldots \qquad (2.4.5)$$

where $\phi(-)$ is the standard normal probability available widely in tabulated form.

2.5 Observations on Reliability Theories:

The failure probability is a function of the probability distributions of R and L. So it is the distribution of the safety margin (R-L) or ln(R/L) that is important in the evaluation of the probability of failure than the distributions of the individual variates. If the risk level is large (say $> 10^{-3}$) the probability distributions are not too important because the calculated failure probability would not be too different regardless of these distributions. However, for very small risks, $< < 10^{-3}$, the failure probability can be quite different depending on these distributions. In this latter case, knowledge of the correct distributions for R and L becomes important. Unfortunately, with limited information, the correct distributions are most difficult to ascertain. In practice, the failure probability may only be calculated on the basis of assumed distributions. Consequently, at very low risk levels, the calculated failure probability is somewhat an ambiguous measure (strictly speaking) of the underlying risk. In spite of this, however, the failure probability remains as the only quantitative measure of risk, and at the very least, a failure probability estimated

on the basis of a prescribed set of distributions remains useful as a consistent (even if only approximate) measure of the underlying risk.

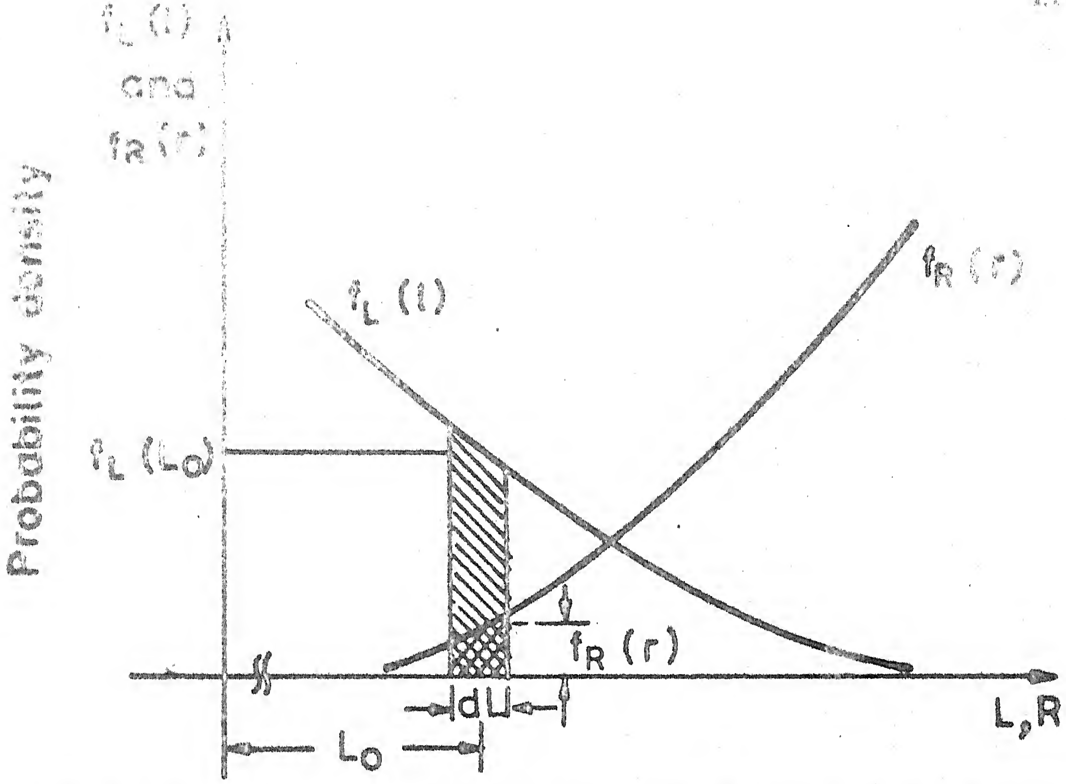


FIG.2·1 COMPUTATION OF FAILURE PROBABILITY FROM THE DISTRIBUTION OF L AND R

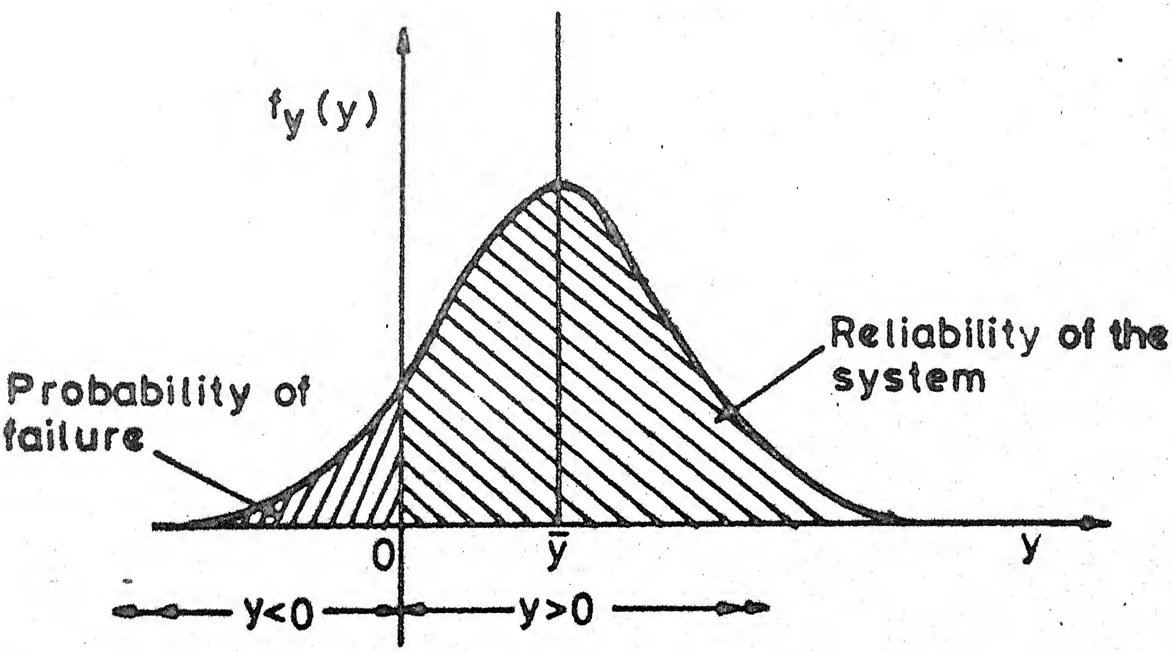


FIG.2·2 DENSITY FUNCTION OF RANDOM VARIABLE y

CHAPTER 3

DESIGN FORMULATION AND DESIGN CHARTS

3.1 General:

This chapter can be broadly divided into two sections. First section deals with the parametric study of the reliability based design of RCC circular slabs. The other section contains formulation of design equations taking into account all the variabilities inherent in the material properties and presentation of design charts for a preassigned probability of failure based on Indian Standard Code of Practice for plain and reinforced concrete[32]. The formulation is kept quite general and the variables are grouped together to reduce their number and bring them into a non dimensional form. The coefficients of variation for different design variables have been taken from the test results on more than 100 slabs compiled by MacGregor and Mirza[22] to keep the design as realistic as possible. Moreover to cover the unknown uncertainities, design equations are formulated taking relative ratios of the coefficient of variation of different design parameters in terms of a variable V. By changing the value of V a different set of coefficients of variation can be considered in the design, although their relative ratios will remain same.

For calculating the external moment on the slabs lowerbound solutions for circular slabs are used. The design charts are presented for both simply supported and fixed edge conditions. For circular slabs with free and fixed edge conditions the upperbound and lowerbound solutions are the same. So it is more appropriate to consider this unique failure mode while calculating external moment. Only the uniformly distributed load is considered.

3.2 Parametric Study of Reliability Based Design of Reinforced Concrete Circular Slabs:

In most of the cases the slabs are usually designed as under-reinforced sections. The basic reliability equation (2.3.8) has been used for investigating the effect of different design parameters on the reliability design of under-reinforced concrete circular slabs. The parameters included in the present study are depth, stress in concrete, stress in steel, area of steel, coefficient of variation and the specified probability of failure.

The basic aim of this study is to find the effect of each parameter on the design qualitatively only keeping all other parameters as constant. This is an investigation from which an educated guess to the combined effect of some of these parameters on the design can be thought of. In a

realistic problem their effect on the design considering the interaction (if any) should be studied to have the over all picture of the problem. However the present study has been used as a preliminary investigation before formulating the reliability design of concrete circular slabs by **clubbing** a number of design parameters together in such a way that the whole design is in non-dimensional form.

Starting with an initial design, increments are given to each parameter turn by turn and the effect on area of reinforcement is found.

3.2.1 Formulation of the Problem:

For a reinforced concrete slab the equation (2.3.8) can be written as

$$Z = \frac{\bar{M}_R - \bar{M}_L}{\sqrt{S^2_{M_R} + S^2_{M_L}}} \qquad \ldots \qquad (3.2.1)$$

where,

$\bar{M}_R$ = Mean expected moment of resistance of the slab per metre width

$\bar{M}_L$ = Mean design moment per metre width of slab

S_{M_R} and S_{M_L} = Standard deviations of $\bar{M}_R$ and $\bar{M}_L$ respectively.

The value of Z can be obtained from the standard normal distribution tables for a given value of P_f

(probability of failure) and vice versa (Appendix A).

Mostly slabs are designed as either balanced or under reinforced sections. Taking the moment of resistance equation from Indian Standard Code of Practice for Plain and Reinforced Concrete (IS: 456-1978) for under reinforced section (Appendix B).

$$M_R = A_{st}\, f_y \left(d - K \frac{A_{st}\, f_y}{f_{ck}\, b} \right) \quad \ldots \qquad (3.2.2)$$

where A_{st} = area of reinforcement

d = depth of the slab

b = metre width of the slab

f_Y = tensile stress of steel

f_{ck} = characteristic strength of concrete

K = 0.774.

Here the partial safety factors for both f_y and f_{ck} are ommitted because the variabilities in the stresses are considered in the form of coefficient of variation. For a circular slab the lower bound solution for external moment with uniformly distributed load is

$$(M_L + m) = \frac{p^* R^2}{6} \quad \ldots \qquad (3.2.3)$$

where R = radius of slab

m = negative moment.

p^* = uniformly distributed load.

Let $m = \lambda M_L$

where λ is the ratio of negative moment to positive moment.

Then equation (3.2.3) can be written as

$$M_L = \frac{p^* R^2}{6(1+\lambda)} \quad \ldots \quad (3.2.4)$$

where p = intensity of uniformly distributed load.

It is obvious from equations (3.2.2) and (3.2.4) that M_R and M_L are the functions of a number of design variables X_i. In all practical cases these variables are uncertain to a certain degree. These uncertainities have a considerable effect on the expected values and the standard deviations of M_R and M_L. The exact statistical distribution of M_R and M_L can be obtained if the exact distribution functions of the variables X_i are known. In practice one can hope to estimate the mean and coefficients of variation of the variables. The underlying probability distribution of the random variables is assumed as Gaussian distribution. The mean and standard deviation of a function can be calculated using the mean and standard deviation of the random variables[31].

$$\bar{Y} = f(\bar{X}_1, \bar{X}_2, \bar{X}_3, \ldots\ldots \bar{X}_n) \quad \ldots \quad (3.2.5)$$

$$S_Y = \left[\sum_{i=1}^{n} \left\{ \frac{\partial f(X)}{\partial X_i} \Bigg|_{(\bar{X}_1, \bar{X}_2 \ldots \bar{X}_n)} \right\}^2 S_{X_i}^2 \right]^{1/2} \quad \ldots \quad (3.2.6)$$

where

Y = function of random variables $X_1, X_2 \ldots . X_n$

S_Y = standard deviation of a function of random variables X_1, $X_2 \ldots . . X_n$.

Bar (–) over the variable indicates the mean of that variable. Here after the bar is dropped for the sake of convenience.

Applying the foregoing equations (3.2.5) and (3.2.6) the mean and standard deviations of the functions M_R and M_L can be derived as follows.

$$M_R = A_{st} f_y \left(d - \frac{K A_{st} f_y}{b f_{ck}} \right) \ldots \quad (3.2.7)$$

$$S_{M_R}^2 = \left(\frac{\partial M_R}{\partial A_{st}}\right)^2 S_{Ast}^2 + \left(\frac{\partial M_R}{\partial f_y}\right)^2 S_{fy}^2 + \left(\frac{\partial M_R}{\partial f_{ck}}\right)^2 S_{fck}^2$$

$$+ \left(\frac{\partial M_R}{\partial d}\right)^2 S_d^2 + \left(\frac{\partial M_R}{\partial b}\right)^2 S_b^2$$

$$= \left(f_y d - \frac{2K A_{st} f_y^2}{f_{ck} b}\right)^2 A_{st}^2 V_{Ast}^2$$

$$+ \left(A_{st} d - \frac{2K A_{st}^2 f_y}{f_{ck} b}\right)^2 f_y^2 V_{fy}^2$$

$$\left(\frac{K A_{st}^2 f_y^2}{f_{ck}^2 b}\right)^2 f_{ck}^2 V_{fck}^2$$

$$+ \; f_y^2 \; A_{st}^2 \; d^2 \; V_d^2$$

$$+ \; \left(\frac{K \; A_{st}^2 \; f_y^2}{f_{ck} \; b^2} \right)^2 \; b^2 \; V_b^2 \qquad \ldots \qquad (3.2.8)$$

where V_{Ast} , V_{fy}, V_{fck}, V_d, V_b are the coefficients of variations (COV) of A_{st}, f_y, f_{ck}, d, b respectively

$$COV = \frac{\text{Standard deviation}}{\text{Mean value}}$$

The standard deviation of M_L can be obtained from

$$M_L = \frac{p^* R^2}{6(1+\nu)} \; b \qquad \ldots \qquad (3.2.9)$$

$$S_{M_L}^2 = M_L^2 \; (4 \; V_R^2 + V_{p*}^2 + V_b^2) \qquad \ldots \qquad (3.2.10)$$

where,

V_R and V_{p*} are the COV's of radius of slab and uniformly distributed load respectively.

Since the behaviour has to be studied qualitatively only, the same value of V will be assumed for all the variables. Thus

$$V_{Ast} = V_{fy} = V_{fck} = V_d = V_b = V_R = V_{p*} = V \;\ldots \qquad (3.2.11)$$

By virtue of equation (3.2.11), equations (3.2.8) and (3.2.10) reduce to

$$S_{M_R}^2 = \left[2V^2 \left(f_y d - \frac{2K \; A_{st} f_y^2}{f_{ck} b} \right)^2 A_{st}^2 + \frac{2V^2 K^2 \; A_{st}^4 \; f_y^4}{f_{ck}^2 \; b^2} + V^2 \; f_y^2 \; A_{st}^2 \; d^2 \right] \qquad \ldots \qquad (3.2.12)$$

$$S_{M_L}^2 = 6 \, V^2 \, M_L^2 \qquad \ldots \qquad (3.2.13)$$

Now substituting equations (3.2.7), (3.2.9), (3.2.12) and (3.2.13) in equation (3.2.1), one gets

$$Z = \frac{\left[f_y \, A_{st} \, d - \dfrac{K \, A_{st}^2 \, f_y^2}{f_{ck} \, b} - M_L \right]}{\left[2V^2 \left(f_y d - \dfrac{2K \, A_{st} f_y^2}{f_{ck} b}\right)^2 A_{st}^2 + 2V^2 \dfrac{K^2 A_{st}^4 f_y^4}{f_{ck}^2 \, b^2} + V^2 \, f_y^2 \, A_{st}^2 \, d^2 + 6 M_L^2 \, V^2 \right]^{1/2}} \quad .. \quad (3.2.14)$$

Simplification of equation (3.2.14) yields the following fourth order polynomial in A_{st}

$$A_{st}^4 \left[\frac{K^2 f_y^4}{f_{ck}^2 \, b^2} \left(10V^2 Z^2 - 1\right) \right] + A_{st}^3 \left[\frac{2Kd \, f_y^3}{f_{ck} b} \left(1 - 4KV^2 \, Z^2\right) \right]$$

$$+ A_{st}^2 \left[f_y^2 d^2 \, (3V^2 Z^2 - 1) - \frac{2K \, f_y^2}{f_{ck} \, b} M_L \right]$$

$$+ A_{st} \left[2 \, f_y d \, M_L \right] + M_L^2 \left[6V^2 Z^2 - 1 \right] = 0$$

A computer programe is written for solving the equation (3.2.15) by bisection method and Newton-Raphson method. The effect of other parameters is studied on area of steel turn by turn while keeping all other parameters constant.

The results are as shown in Table II and III. Area of steel versus other design parameters is indicated in Figs. 3.1 and 3.2.

Starting design parameters:

Load = 6.0 kN/m^2

Radius of slab = 5 m

Depth of slab = 100 mm

f_y = 250 MPa

f_{ck} = 15 MPa

The slab considered is fixed along the edge with

ν = 1.0 ; V = 0.05 ; Z = 1.0.

TABLE II

EFFECT OF 'f_{ck}' AND 'f_y' ON AREA OF STEEL REINFORCEMENT

f_{ck} (MPa)	Area of Steel (mm^2)	Percentate Decrease in Area of Steel	f_y (MPa)	Area of Steel (mm^2)	Percentage Decrease in Area of Steel
15.0	630.82	0.00	250.00	630.819	0.00
20.0	616.05	2.34	415.00	380.011	39.76
25.0	607.81	3.64	500.00	315.41	50.00
30.0	602.66	4.46			
35.0	599.05	5.04			

TABLE III

EFFECT OF 'V', 'Z', 'd' ON AREA OF STEEL REINFORCEMENT

V	Area of Steel A_{st} (mm^2)	Percentage Increase in A_{st}	Z	Area of Steel A_{st} (mm^2)	Percentage Increase in A_{st}	d (mm)	A_{st} (mm^2)	Percentage Decrease in A_{st}
0.050	630.82	0.00	1.0	630.82	0.00	100	630.82	0.00
0.075	683.15	8.29	2.0	740.23	17.34	110	564.01	10.59
0.100	740.20	17.34	3.0	873.14	38.41	120	510.80	19.02
0.125	803.09	27.31	4.0	1043.42	65.40	130	467.25	25.93
0.150	873.14	38.41	5.0	1282.36	103.29	140	430.85	31.70

The following observations are made from the above study:

i) The compressive strength of concrete has no significant effect on the area of steel. Using higher grade concrete will result in only nominal decrease in the area of steel. The curve shown in Fig. 3.1 A_{st} Vs f_{ck} is almost flat.

ii) With the increase in tensile strength of steel the required area of steel is decreased considerably. Almost 50 percent of the steel area is decreased with increase in grade of steel from 250 MPa to 500 MPa.

iii) The reliability parameter (i.e. probability of failure) is found to be more sensitive. With increasing values of Z the area of steel increases rapidly.

iv) The coefficient of variation is also a sensitive parameter. The increase in coefficient of variation results in increase of steel area.

v) With the increase in depth of the slab the area of steel decreases. At greater depths the percentage decrease in steel area is some what less than what it is at smaller depths.

3.3 Reliability Based Design of RCC Circular Slabs with Uncurtailed Reinforcement:

Based on the classical reliability approach, an attempt has been made in the present work to formulate the design concepts for under-reinforced concrete circular slabs in flexure only and prepare design charts for the same. To cover the effect of unknown uncertainities the coefficient of variation (COV) is also taken as one of the variables, although their relative ratios have been maintained for all values of COV considered on the problem. The slabs are assumed to be with isotropic reinforcement, which is most common in circular slabs. The design for shear strength is not considered in the present work, because in most of the designs for RCC slabs shear is not critical.

In the next section the formulation of equations is presented. These equations will be similar to those developed

in case of parametric study. But here all the design variables are converted into non-dimensionalized form, from which an equation of fourth order polynomial in terms of steel area is deduced.

3.3.1 Formulation of the Design Problem:

Usually the slabs will be designed as under-reinforced sections and their moment capacity as per Indian Standard Code of Practice for plain and reinforced concrete[32] can be obtained as follows (Appendix B):

$$M_R = A_{st} f_y \left(d - \frac{K A_{st} f_y}{b f_{ck}}\right) \qquad \ldots \quad (3.3.1)$$

where $K = 0.774$

Balanced steel area is obtained from

$$\frac{X_{u,max}}{d} = \frac{A_{st} f_y}{0.5423 f_{ck} db} \qquad \ldots \quad (3.3.2)$$

where $X_{u,max}$ is maximum permissible neutral axis depth

and $$\frac{X_{u,max}}{d} = \frac{0.0035}{0.0055 + \frac{f_y}{E_s}} \qquad \ldots \quad (3.3.3)$$

from equations (3.3.2) and (3.3.3)

$$p_o = \frac{0.0035}{0.0055 + f_y/E_s} \times \frac{54.23 f_{ck}}{f_y} \qquad \ldots \quad (3.3.4)$$

p_o = Percentage of balanced reinforcement.

From the above equation one can see that the balanced area of steel reinforcement will depend on grade of steel and concrete.

$$p_o = \frac{A_{st}}{bd} \times 100$$

Dividing equation (3.3.1) by a quantity $bd^2\ f_{ck}$ throughout, a non-dimensional expression is obtained as

$$\frac{M_R}{bd^2\ f_{ck}} = \frac{A_{st}}{bd}\ \frac{f_y}{f_{ck}}\ (1 - K\ \frac{A_{st}}{bd}\ \frac{f_y}{f_{ck}}\ ) \quad \ldots \quad (3.3.5)$$

Taking

$$Q_R = \frac{M_R}{bd^2\ f_{ck}} \quad \ldots \quad (3.3.6)$$

$$p = \frac{A_{st}}{bd} \times 100 \quad \ldots \quad (3.3.7)$$

$$m = \frac{f_y}{100\ f_{ck}} \quad \ldots \quad (3.3.8)$$

Now equation (3.3.2) can be written as

$$Q_R = pm\ (1 - K\ pm) \quad \ldots \quad (3.3.9)$$

where, p has been expressed as percentage of steel.

The external moment for uniformly distributed load on the circular slab is given by the following expression

$$M_L = \frac{p^* R^2}{6(1+\nu)} \quad \ldots \quad (3.3.10)$$

where p^* = uniformly distributed load.

Now converting the above equation into a non-dimensionalized form by dividing the equation with $d^2 f_{ck}$ through out.

$$\frac{M_L}{d^2 f_{ck}} = \frac{p^* R^2}{6(1+\nu)d^2 f_{ck}} \quad \ldots \quad (3.3.11)$$

$$Q_L = \left(\frac{p^*}{f_{ck}}\right)\left(\frac{R}{d}\right)^2 \frac{1}{6(1+\nu)} \quad \ldots \quad (3.3.12)$$

$$Q_L = PL^2 / 6(1+\nu) \quad \ldots \quad (3.3.13)$$

where $Q_L = M_L/(d^2 f_{ck})$ .. (3.3.14)

$$P = p^*/f_{ck} \quad \ldots \quad (3.3.15)$$

$$L = R/d \quad \ldots \quad (3.3.16)$$

Now assuming Q_R, Q_L as the normal variates, the reliability parameter 'Z' from equation (3.2.1) is

$$Z = \frac{\bar{Q}_R - \bar{Q}_L}{(S^2_{Q_R} + S^2_{Q_L})^{1/2}} \quad \ldots \quad (3.3.17)$$

where Q_R and Q_L are functions of several variables.

$$Q_R = f(d, f_{ck}, f_y, A_{st}, b)$$

$$Q_L = f(p^*, R, d, f_{ck})$$

Now assuming V_{p^*}, V_R, V_d, $V_{f_{ck}}$, V_{f_y}, $V_{A_{st}}$, V_b as the respective coefficients of variation for variables p^*, R, d, f_{ck}, A_{st}, b and using the equations (3.2.5), (3.2.6), (3.3.1) through (3.3.17) the following expressions for standard deviations of load and resistance can be obtained.

$$S^2_{Q_R} = \left(\frac{\partial Q_R}{\partial p}\right)^2 S^2_p + \left(\frac{\partial Q_R}{\partial m}\right)^2 S^2_m \quad \ldots \quad (3.3.18)$$

$$S_{Q_L}^2 = \left(\frac{\partial Q_L}{\partial P}\right)^2 S_P^2 + \left(\frac{\partial Q_L}{\partial L}\right)^2 S_L^2 \quad \ldots \quad (3.3.19)$$

$$\left(\frac{\partial Q_R}{\partial p}\right)^2 = (m\ (1-2K\ pm))^2 \quad \ldots \quad (3.3.20)$$

$$\left(\frac{\partial Q_R}{\partial m}\right)^2 = (p - 2\ Km\ p^2)^2 \quad \ldots \quad (3.3.21)$$

$$S_p^2 = \left(\frac{\partial p}{\partial A_{st}}\right)^2 S_{A_{st}}^2 + \left(\frac{\partial p}{\partial b}\right)^2 S_b^2 + \left(\frac{\partial p}{\partial d}\right)^2 S_d^2 \ldots$$

$$= p^2(V_{A_{st}}^2 + V_b^2 + V_d^2) \quad \ldots \quad (3.3.22)$$

$$S_m^2 = \left(\frac{\partial m}{\partial f_y}\right)^2 S_{f_y}^2 + \left(\frac{\partial m}{\partial\ f_{ck}}\right)^2 S_{f_{ck}}^2 \ldots$$

$$= m^2\ V_{f_y}^2 + m^2\ V_{f_{ck}}^2$$

$$= m^2\ (V_{f_y}^2 + V_{f_{ck}}^2) \quad \ldots \quad (3.3.23)$$

$$S_P^2 = \left(\frac{\partial P}{\partial p*}\right)^2 S_{p*}^2 + \left(\frac{\partial P}{\partial f_{ck}}\right)^2 S_{f_{ck}}^2 \quad \ldots \quad (3.3.24)$$

$$= P^2\ (V_{f_{ck}}^2 + V_{p*}^2) \quad \ldots \quad (3.3.25)$$

$$S_L^2 = \left(\frac{\partial L}{\partial R}\right)^2 S_R^2 + \left(\frac{\partial L}{\partial d}\right)^2 S_d^2 \quad \ldots \quad (3.3.26)$$

$$= \left(\frac{R}{d}\right)^2 (V_R^2 + V_d^2) \quad \ldots \quad (3.3.27)$$

substituting equations (3.3.22) and (3.3.23) in equation (3.3.18)

$$S_{Q_R}^2 = p^2\left[m^2(1-2K\ pm)^2\right](V_{A_{st}}^2 + V_b^2 + V_d^2)$$
$$+ m^2\left[p^2(1-2K\ pm)^2\right](V_{f_y}^2 + V_{f_{ck}}^2)$$

$$S^2_{Q_R} = (p^2m^2 + 4K^2p^4m^4 - 4K\ p^3m^3)\ x$$

$$(V^2_{A_{st}} + V^2_b + V^2_d + V^2_{f_y} + V^2_{f_{ck}}) \quad \ldots \quad (3.3.28)$$

Substituting equations (3.3.25) and (3.3.27) in equation (3.3.19)

$$S^2_{M_L} = \frac{L^4}{36(1+\nu)^2}\ P^2(V^2_{p*} + V^2_{f_y}) + \frac{4P^2\ L^2}{36(1+\nu)^2}$$

$$L^2\ (V^2_R + V^2_d$$

$$S^2_{M_L} = (V^2_{p*} + V^2_{f_{ck}} + 4V^2_R + 4\ V^2_d)\ \frac{P^2L^4}{36(1+\nu)^2}$$

$$\ldots \quad (3.3.29)$$

COV's for different variables using the tests results on slabs by MacGregor and Ellingwood(22)

$V_{f_{ck}} = 0.18$ $\quad V_{f_y} = 0.107$

$V_{A_{st}} = 0.10$ $\quad V_{p*} = 0.12$

$V_d = 0.017$ $\quad V_b = 0.01$

$V_R = 0.01$ $\quad V_{E_s} = 0.033$ (COV of Elastic Modulus)

... (3.3.30)

Assuming the relative ratios of the COV's in (3.3.30) and still treating it as a variability in V , they may be expressed as

$$V_{f_{ck}} = 18\ V\ ;\ V_{f_y} = 10.7\ V\ ;\ V_{A_{st}} = 10\ V$$

$$V_{p*} = 12\ V\ ;\ V_d = 1.7\ V\ ;\ V_R = V;\ V_b = V$$

$$\ldots \quad (3.3.31)$$

Then from equation (3.3.30) and (3.3.31)

$$S_{M_R}^2 = (p^2m^2 + 4K^2p^4m^4 - 4Kp^3m^3)$$

$$((10V)^2 + V^2 + (1.7V)^2 + (10.7V)^2 + (18V)^2)$$

$$= 542.38\ V^2(p^2m^2 + 4K^2p^4m^4 - 4Kp^3m^3) \quad \ldots (3.3.32)$$

$$S_{M_L}^2 = \frac{P^2L^4}{36(1+\nu)^2}\ ((12V)^2 + (18V)^2 + 4V^2 + 4(1.7V)^2)$$

$$= 483.56\ V^2\ \frac{P^2L^4}{36(1+\nu)^2} \quad \ldots (3.3.33)$$

Now substituting the equations (3.3.9), (3.3.13), (3.3.32) and (3.3.33) in equation (3.3.17)

$$Z = \frac{pm\ (1 - K\ pm\ ) - PL^2/\ 6(1+\nu)}{\left[542.38V^2(p^2m^2+4K^2p^4m^4-4Kp^3m^3)+483.56V^2P^2L^4/36(1+\nu)^2\right]^{1/2}} \quad \ldots (5.3.34)$$

Simplying the above equation

$$Z^2\ (542.38V^2\ (p^2m^2+4K^2p^4m^4-4Kp^3m^3) + 483.56V^2P^2L^4\ /\ 36\ (1\ +\nu)^2)$$

$$= pm - K\ p^2m^2 - PL^2/6(1+\nu)^2$$

further simplication yields

$$p^4\ (2169.52V^2K^2Z^2m^4 - K^2m^4) + p^3(2Km^3-2169.52V^2Z^2Km^3)$$

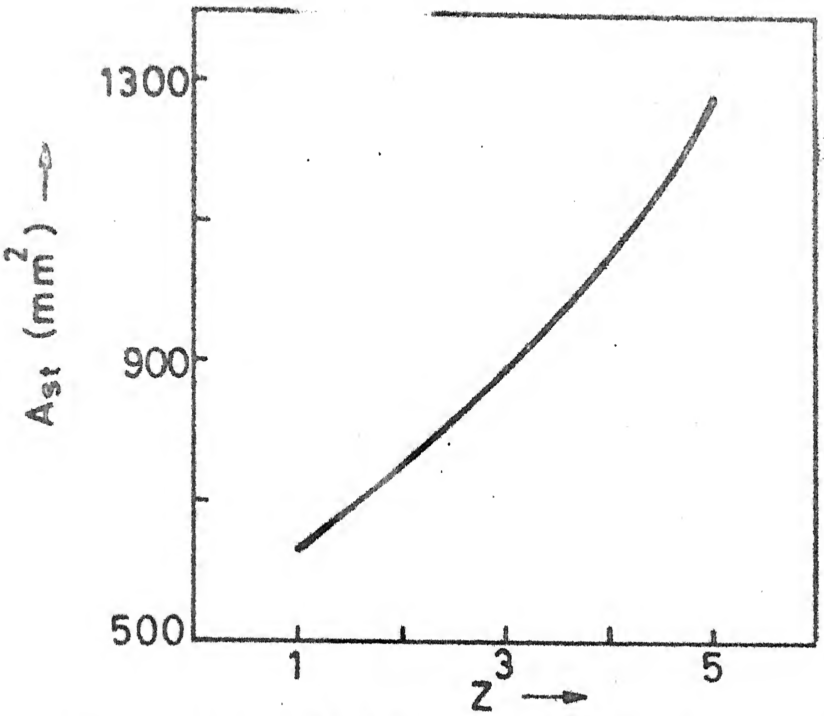
$$+ p^2(542.38V^2Z^2m^2-m^2 - Km^2PL^2/3(1+\nu))$$

$$+ p\ (\frac{PL^2m}{3(1+\nu)}\ ) + \frac{P^2L^4}{36(1+\nu)^2}\ (483.56V^2Z^2-1) = 0 \quad \ldots (3.3.35)$$

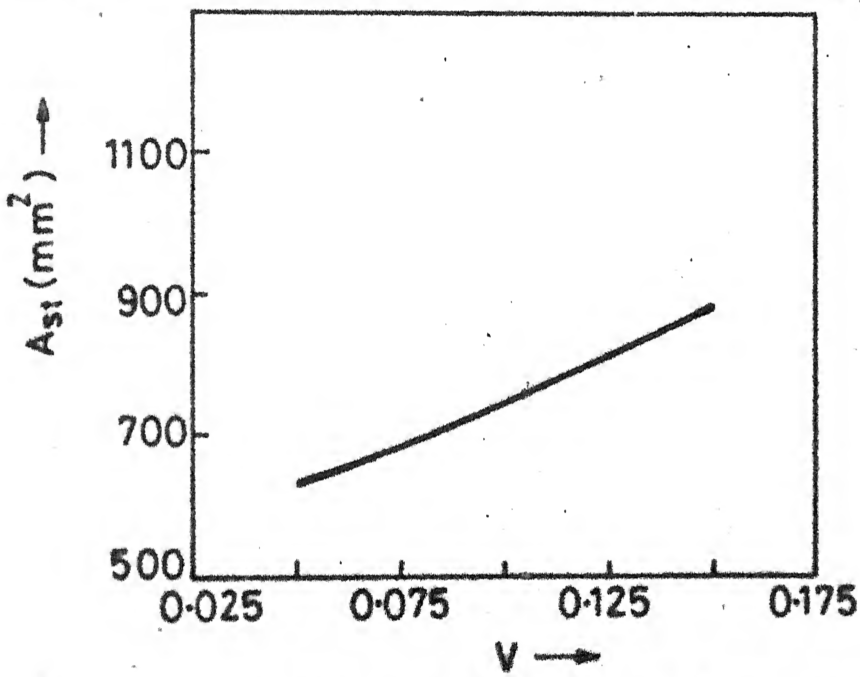
The above equation (3.3.35) is a 4th order polynomial in p (percentage of steel) and is of the same form as that of equation (3.2.1.4). It can be solved for p (percentage of steel) for different values of m, P, L, V, Z. It can be noted from the above equation (3.3.35) that the number of variables are decreased to four from seven in the case of equation (3.2.14). Moreover by simply changing the value of K from 0.776 to 0.91 design charts for IS 456:1964 can be produced. And from changing the value of K from 0.776 to 0.59, the above equation can be used for the design of slabs according to American Code of practice.

The above equation (3.3.35) is formulated on the basis of equation from Indian Standard Code of Practice for plain and reinforced concrete[32]. The design is restricted to isotropic reinforcement and inflexure only. So the values of p obtained from the above equation (3.3.35) will be the reinforcement based on limit state of strength in flexure only. Design charts are presented from the same equation (3.3.35) for different values of f_y, f_{ck}, V, Z. The graphs are drawn for P versus p. p*is taken equal to a minimum of 3 kN/m^2 to a maximum of 12 kN/m^2. The load range on slabs generally will vary between these values only. The percentage of steel is restricted to balanced percentage of reinforcement for different values of f_y and f_{ck}. Some

charts are presented for both simply supported RCC slabs and some for fixed edge conditions. Knowing the value of (R/d) and load p* one can directly pick up the value of P (percentage of reinforcement) from the appropriate chart. The use of charts is illustrated in the final chapter by solving two examples, one each from the above mentioned two edge conditions at the boundary.

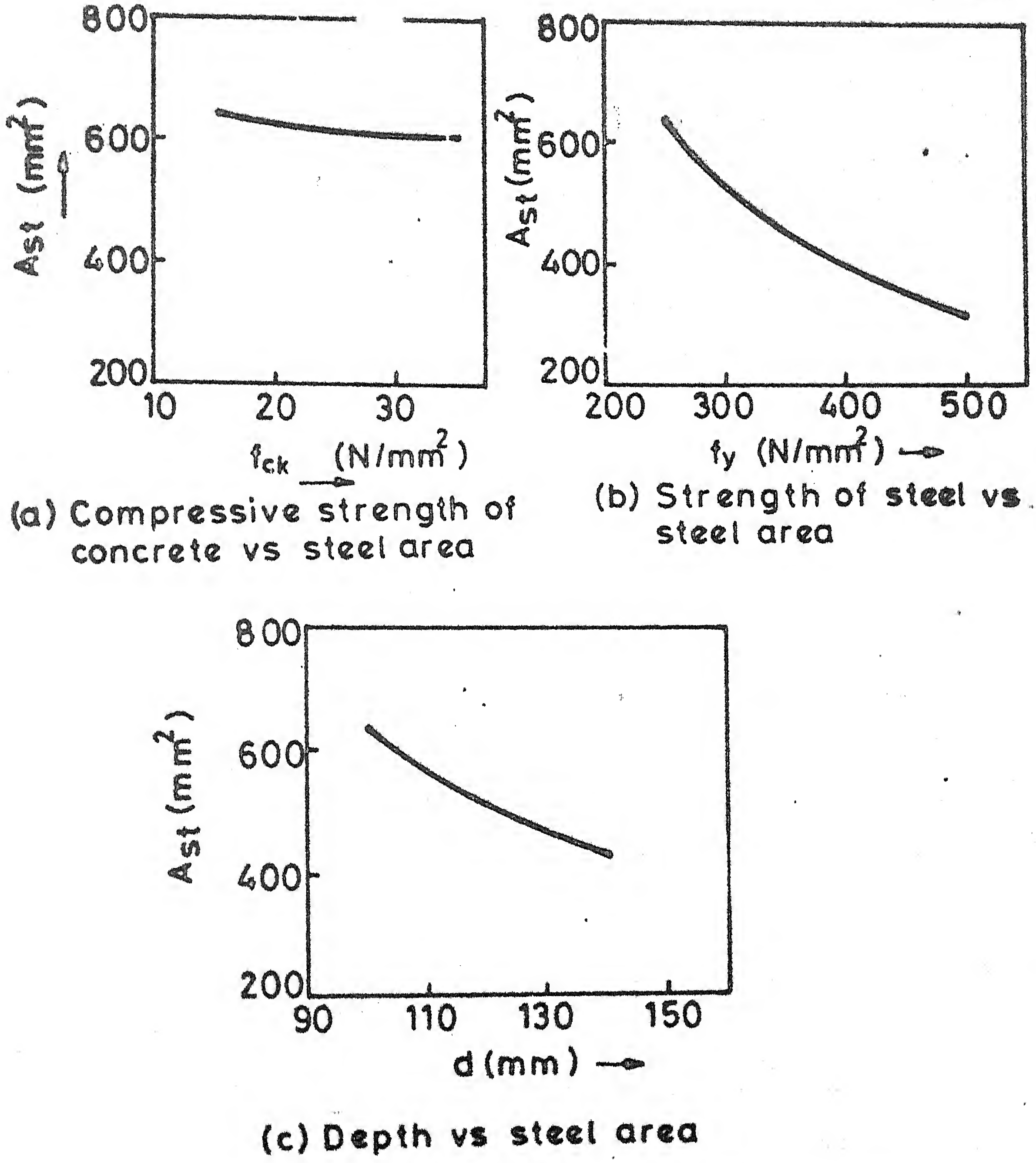


(a) Variation of steel area with reliability parameter



(b) Variation of steel area with coefficient of variation

FIG. 3·1 VARIATION OF STEEL WITH OTHER DESIGN PARAMETERS

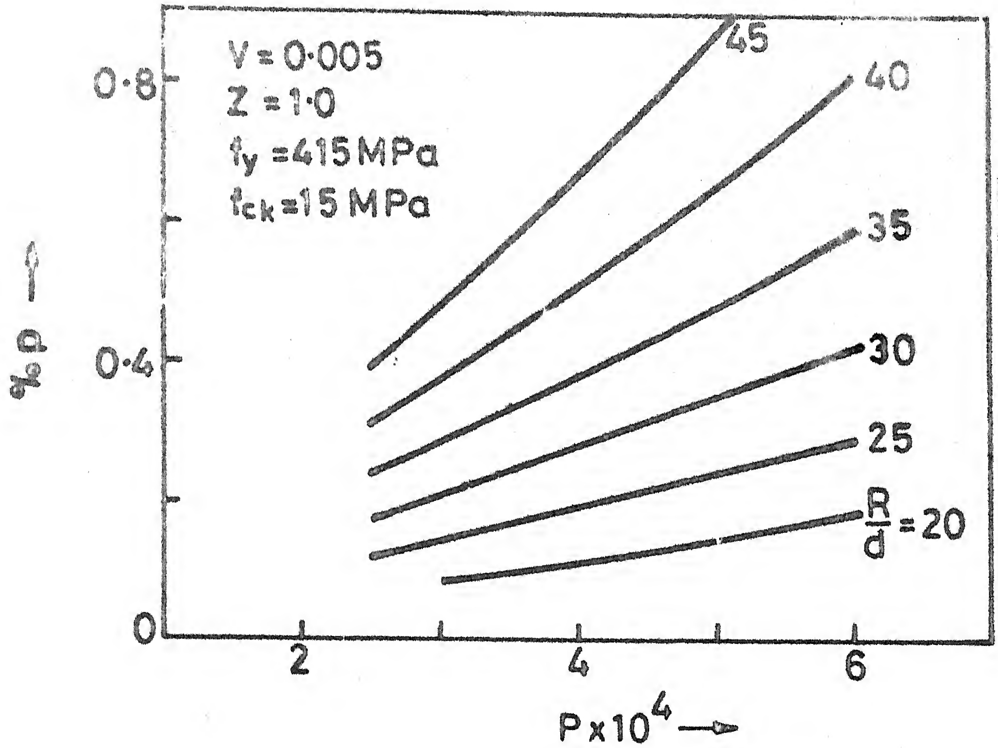


(a) Compressive strength of concrete vs steel area

(b) Strength of steel vs steel area

(c) Depth vs steel area

FIG. 3.2 VARIATION OF STEEL AREA WITH OTHER DESIGN PARAMETER



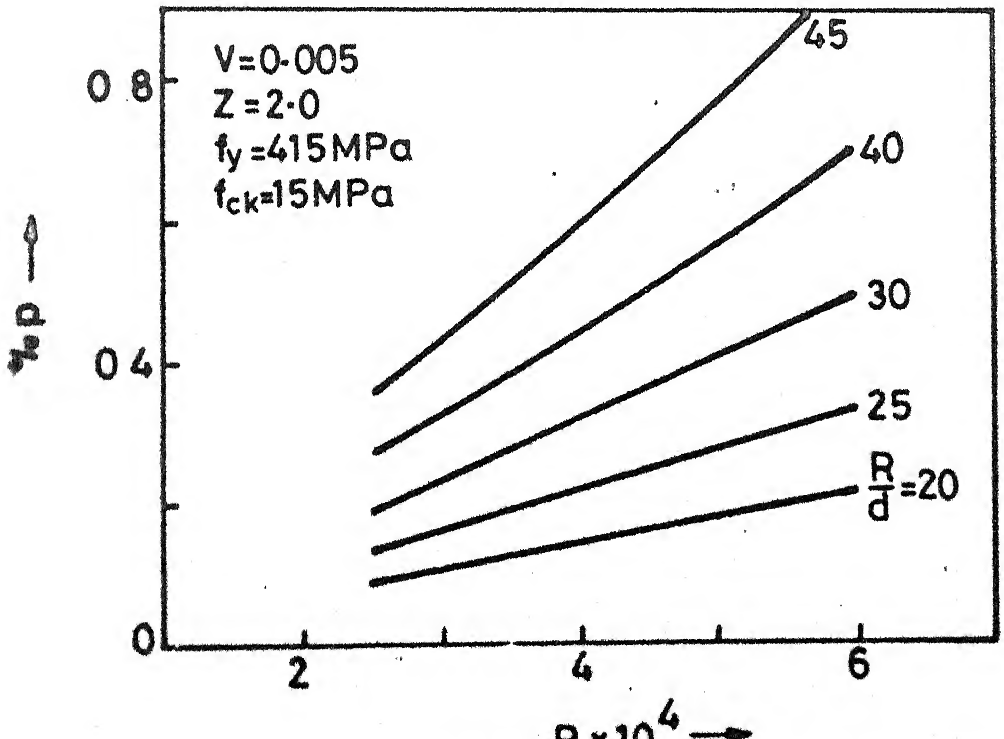
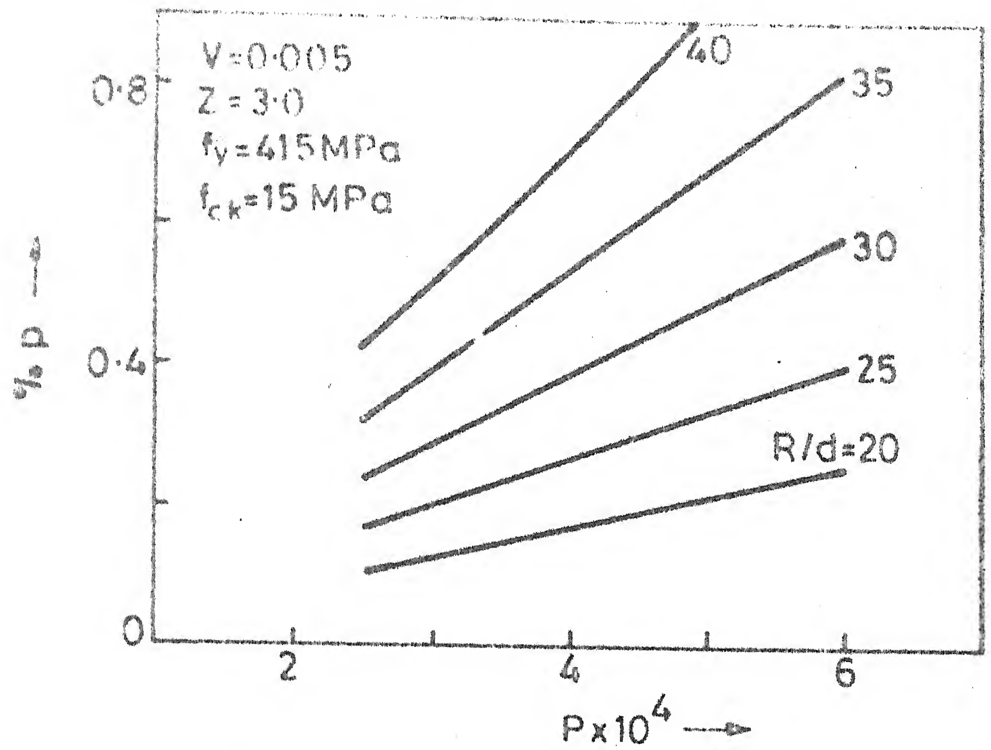


FIG. 3·3 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS



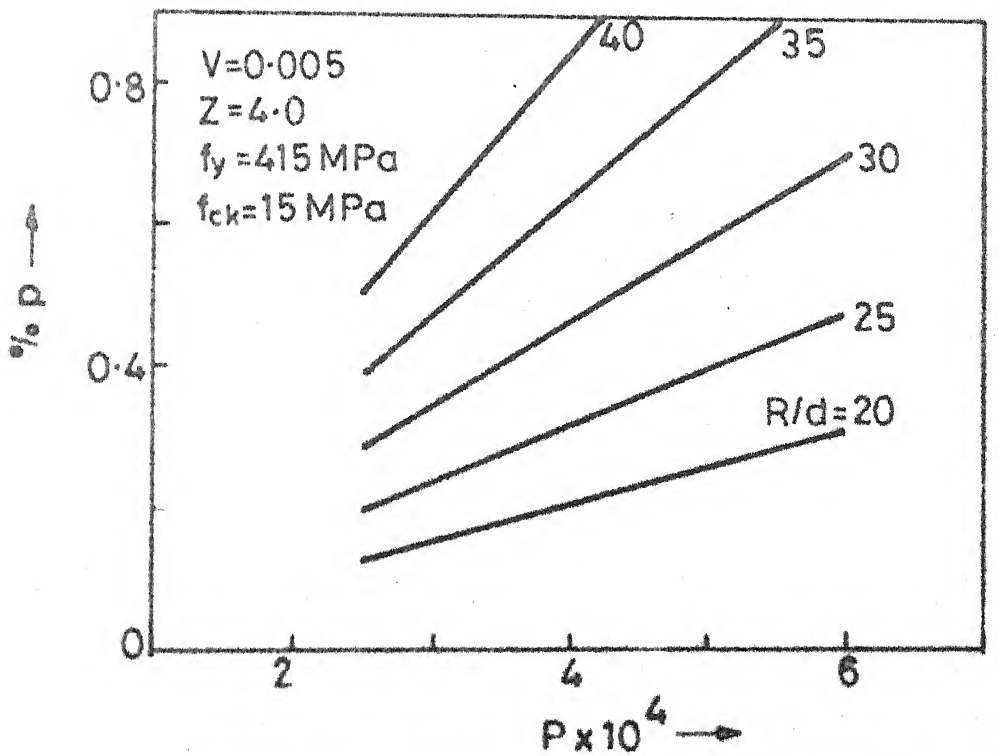


FIG.3·4 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS

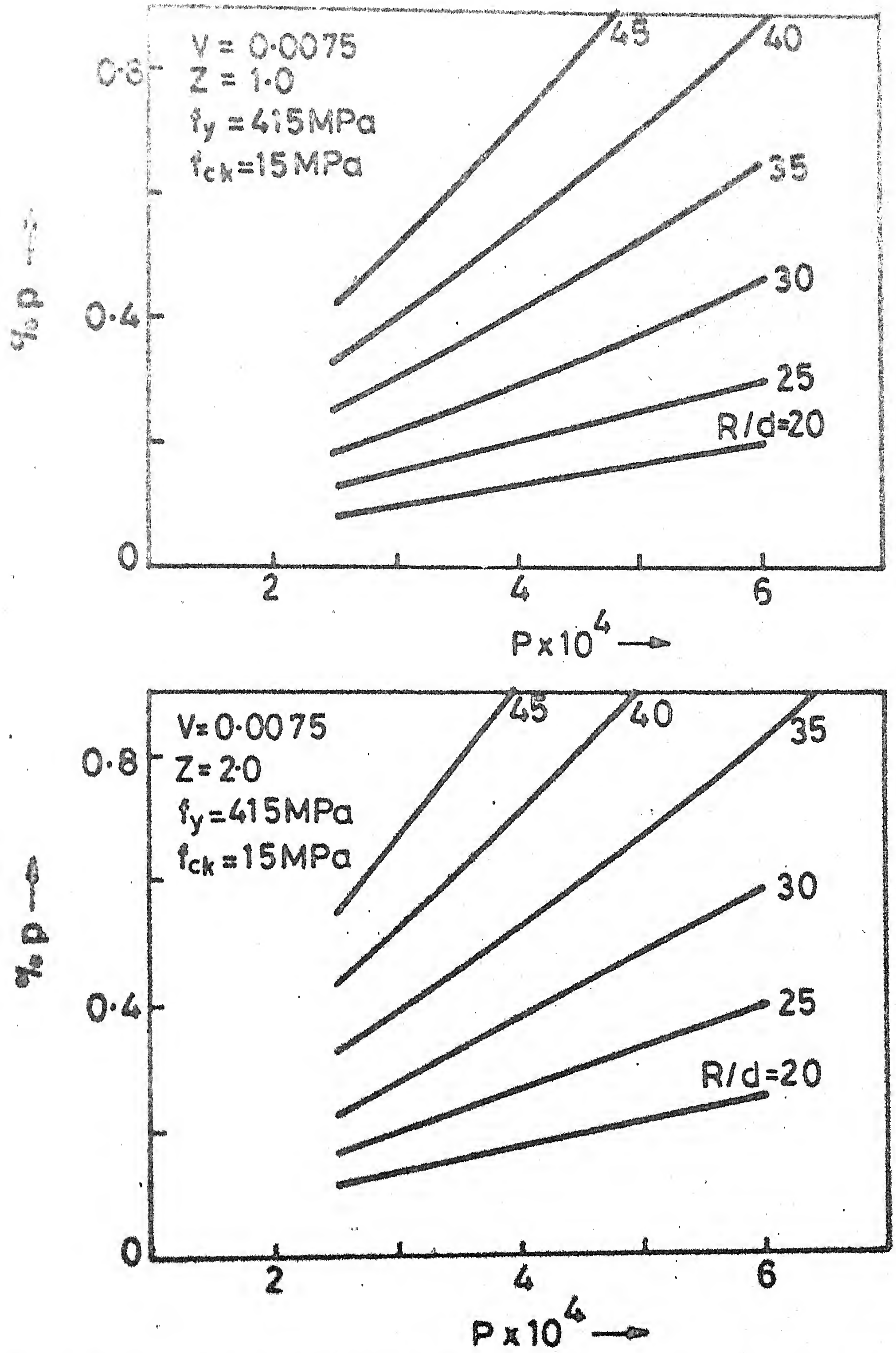


FIG. 3·5 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS

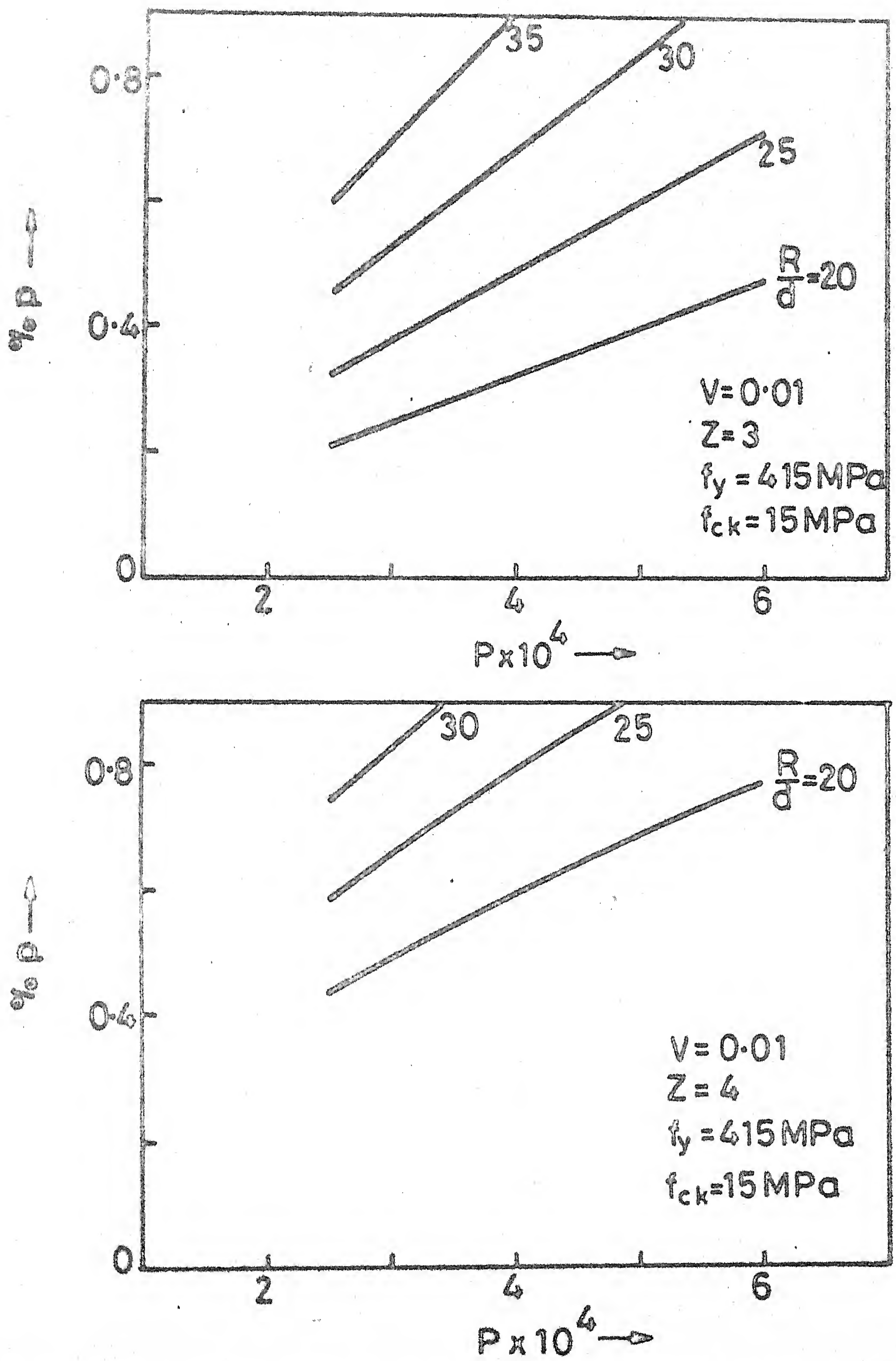


FIG. 3·8 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS

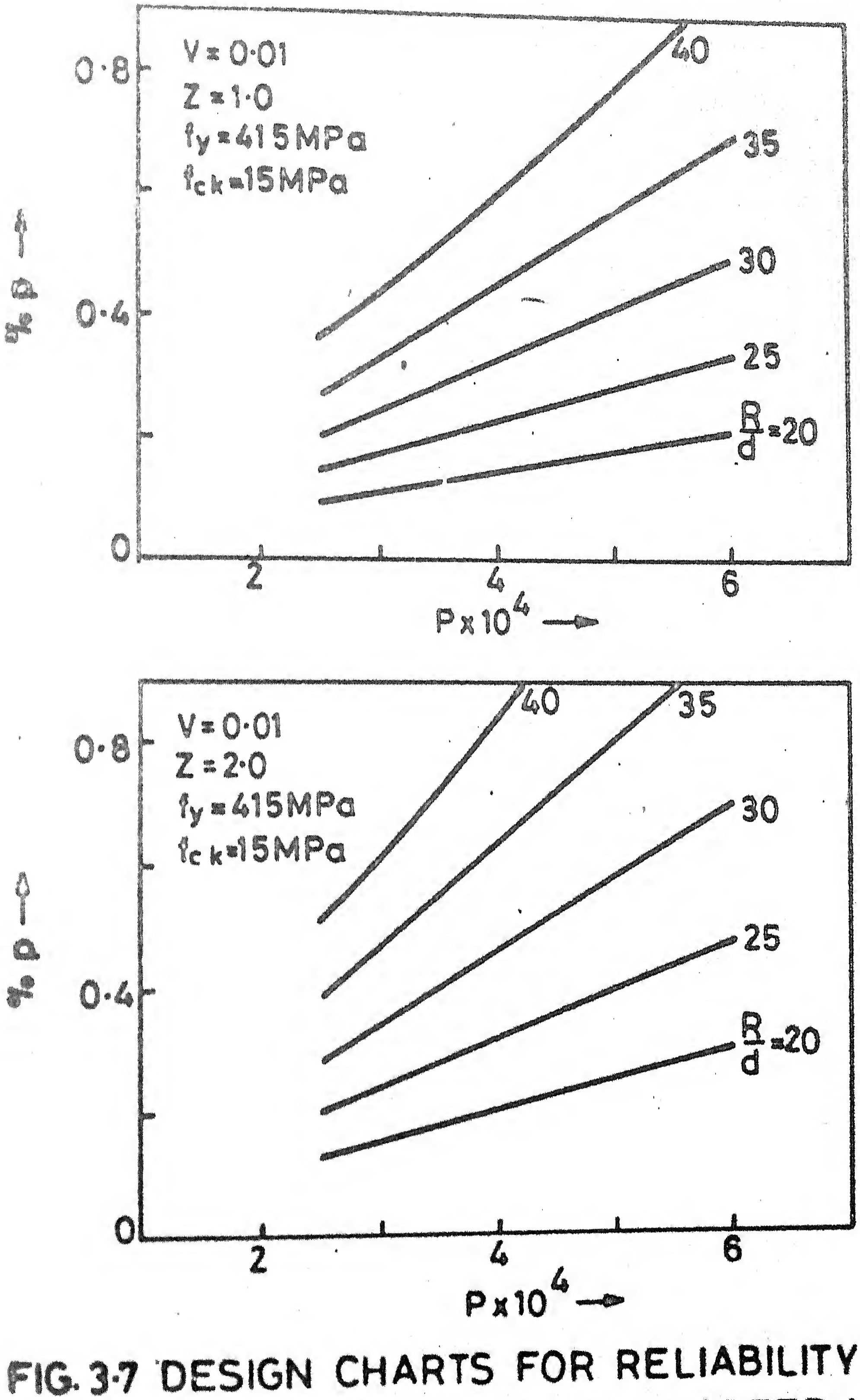


FIG. 3·7 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER REINFORCED CONCRETE CIRCULAR SLABS

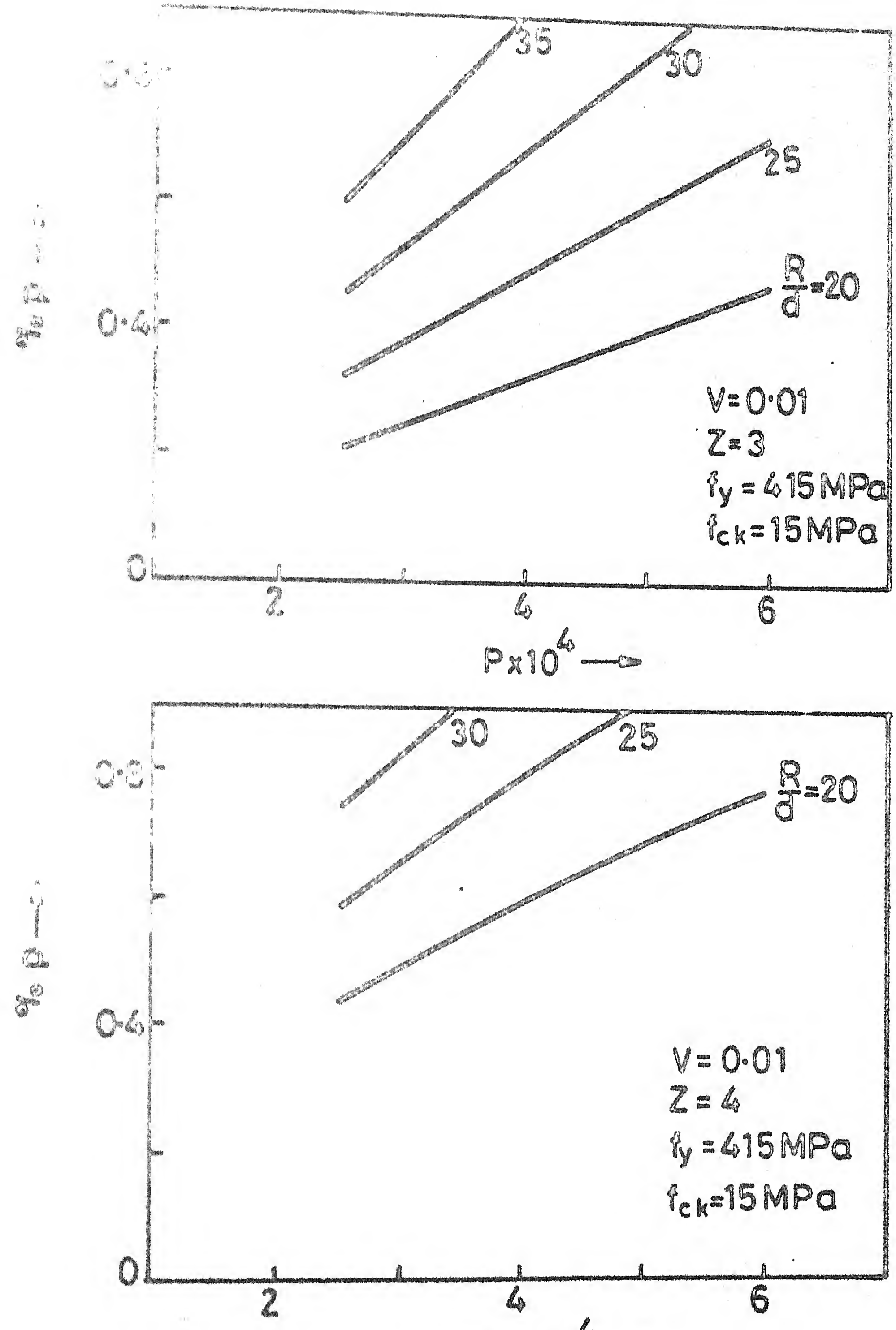
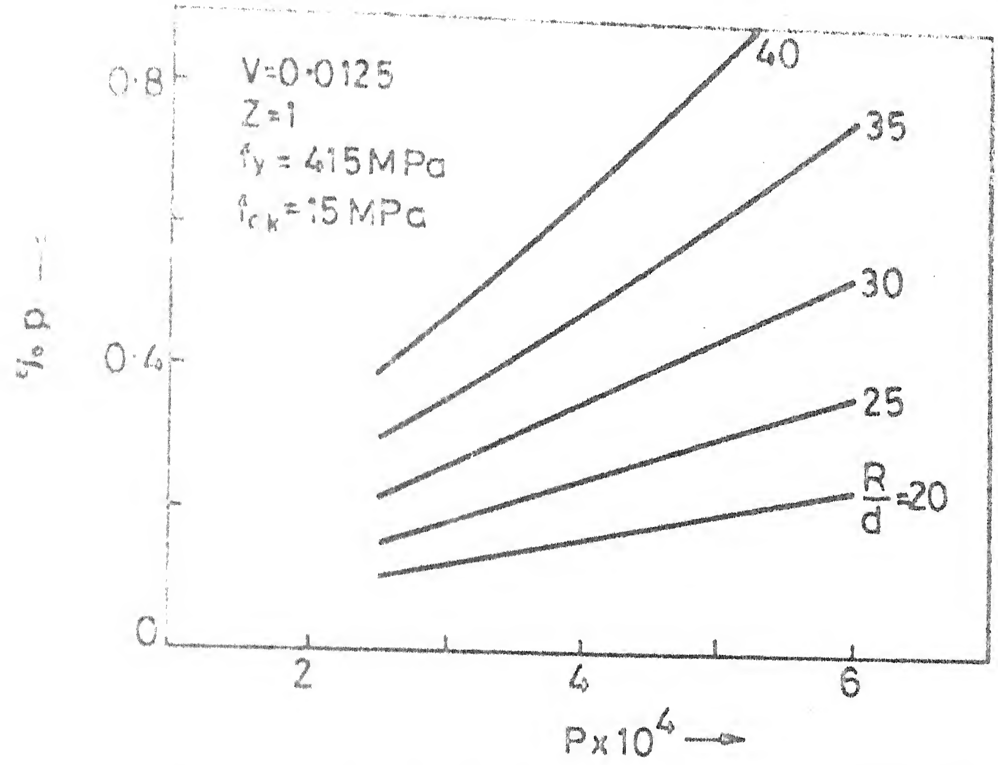


FIG. 3·8 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS



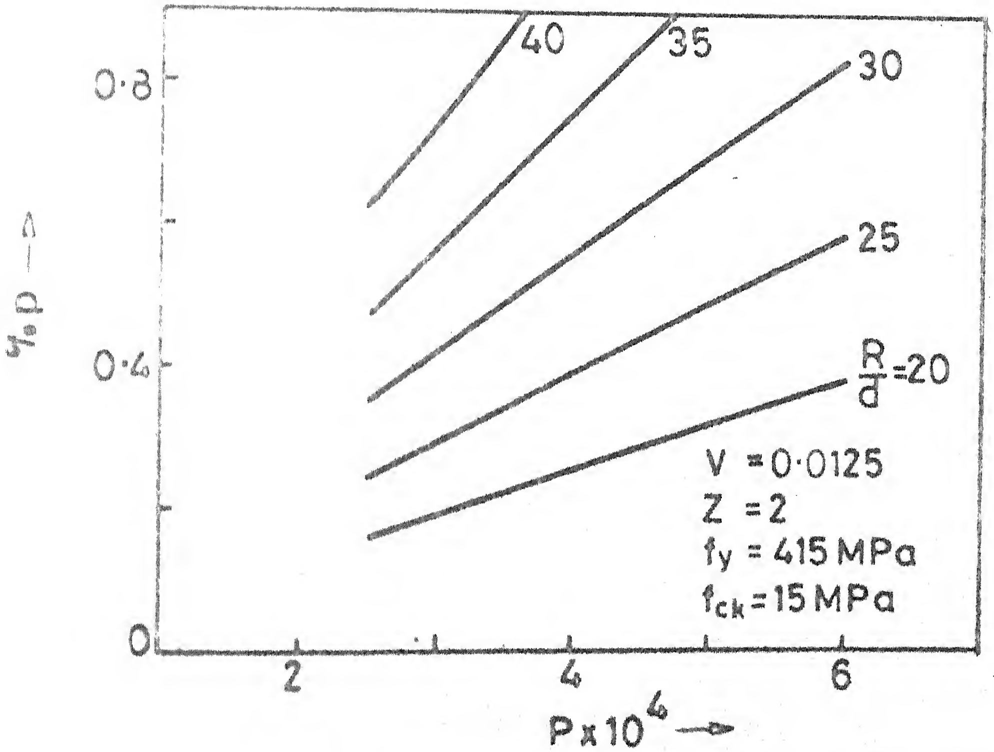


FIG. 3.9 DESIGN CHARTS FOR RELIABILITY DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS

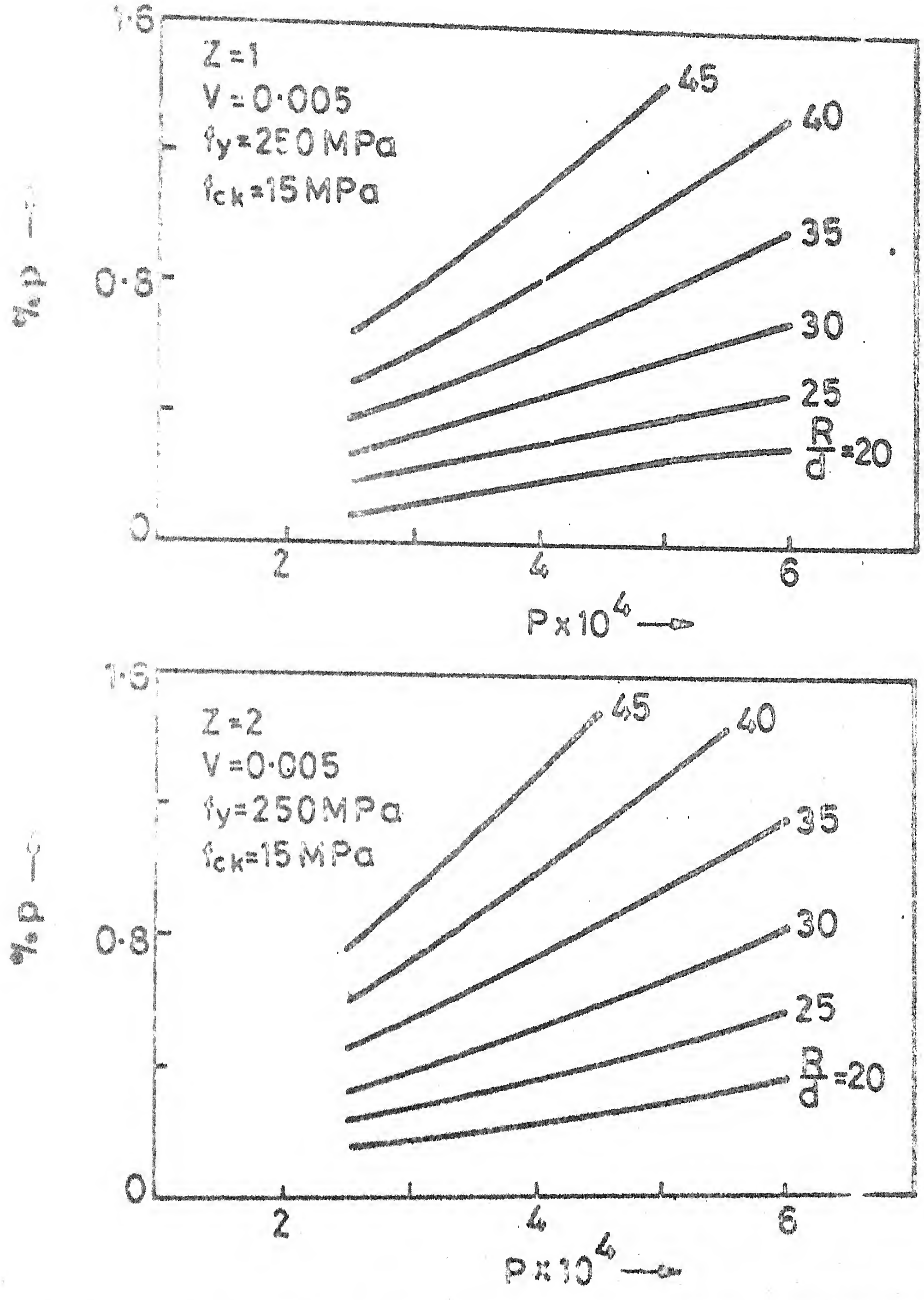


FIG. 3·10 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS

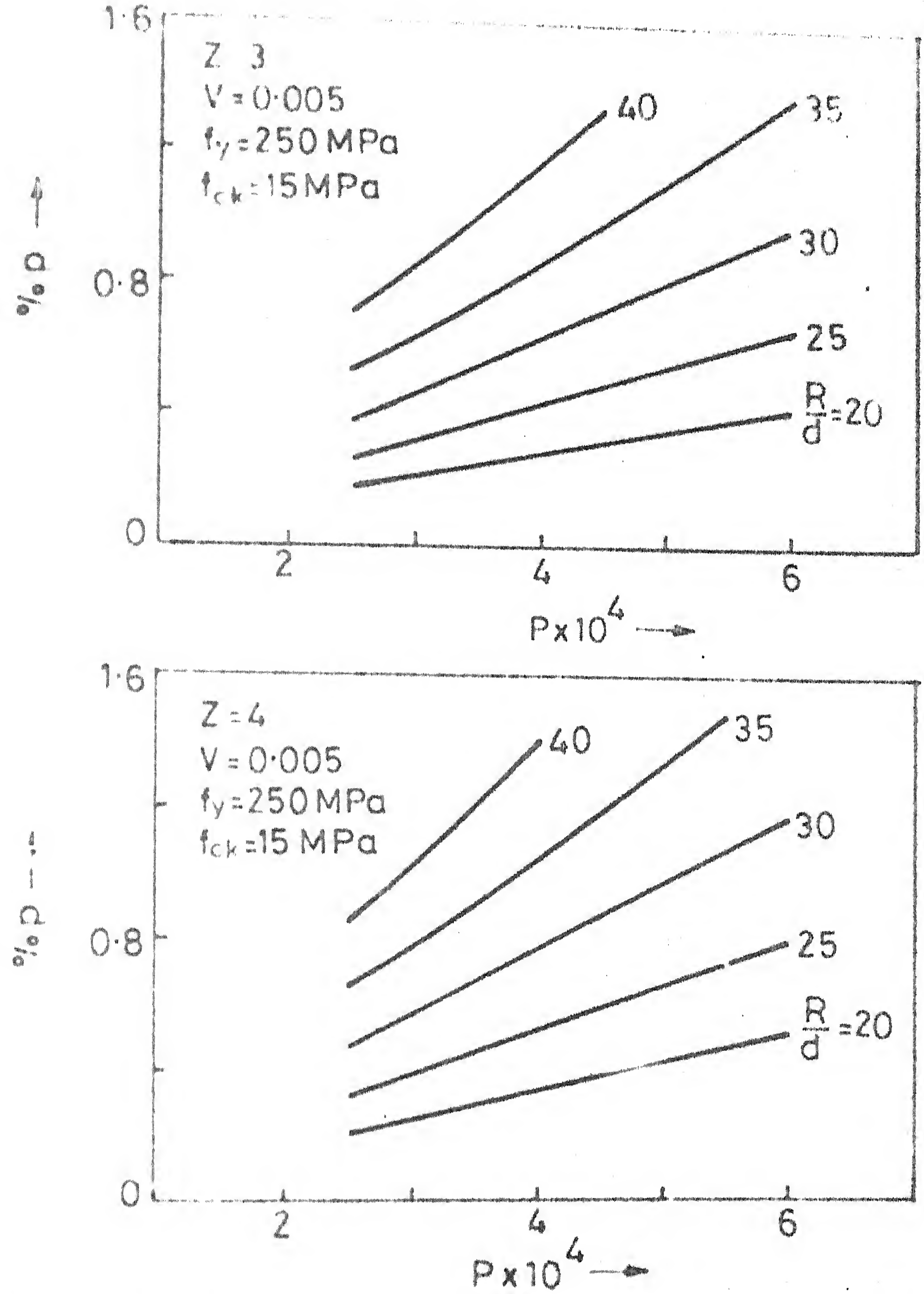


FIG. 3.11 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS

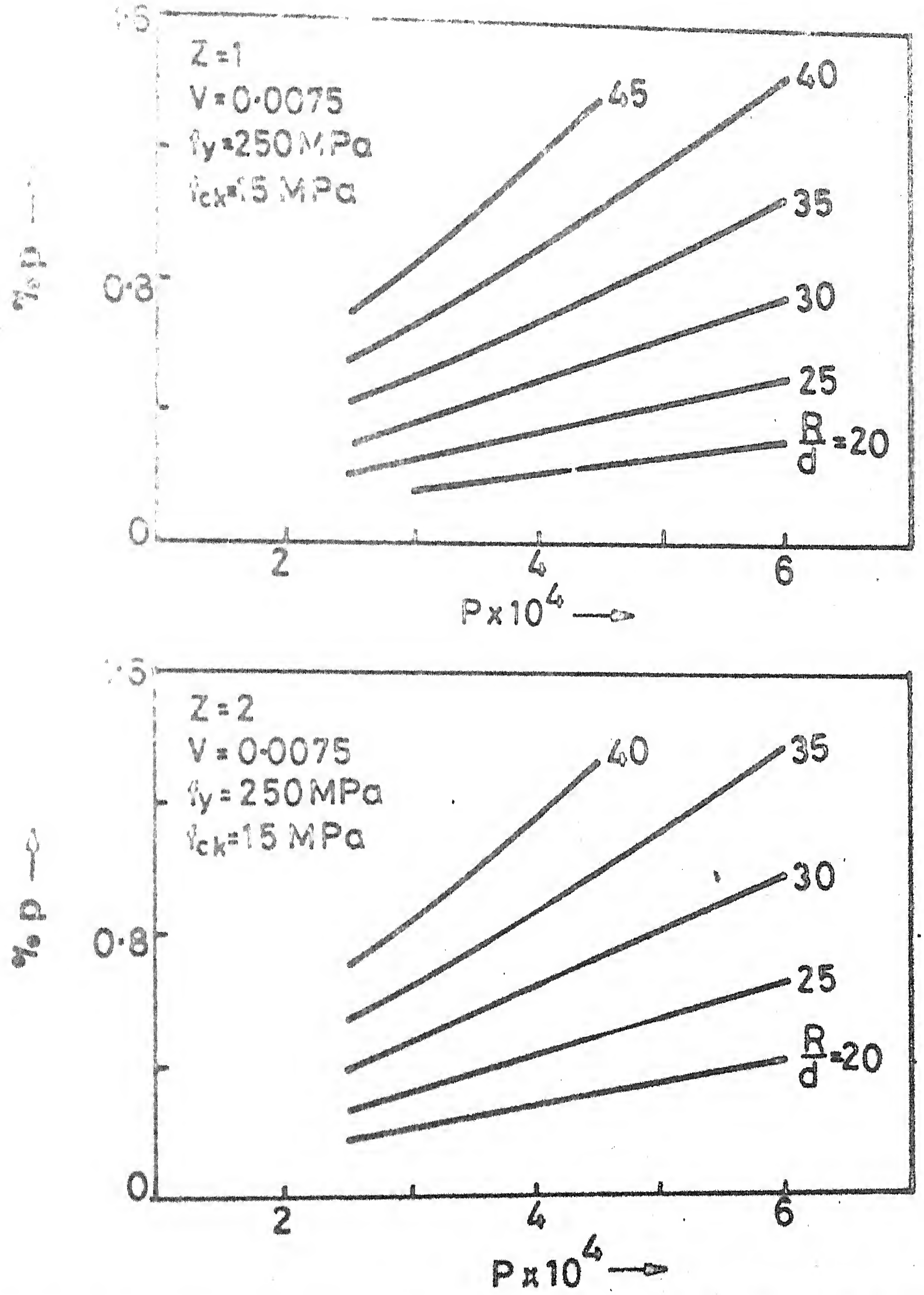


FIG. 3·12 DESIGN CHARTS FOR STABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS

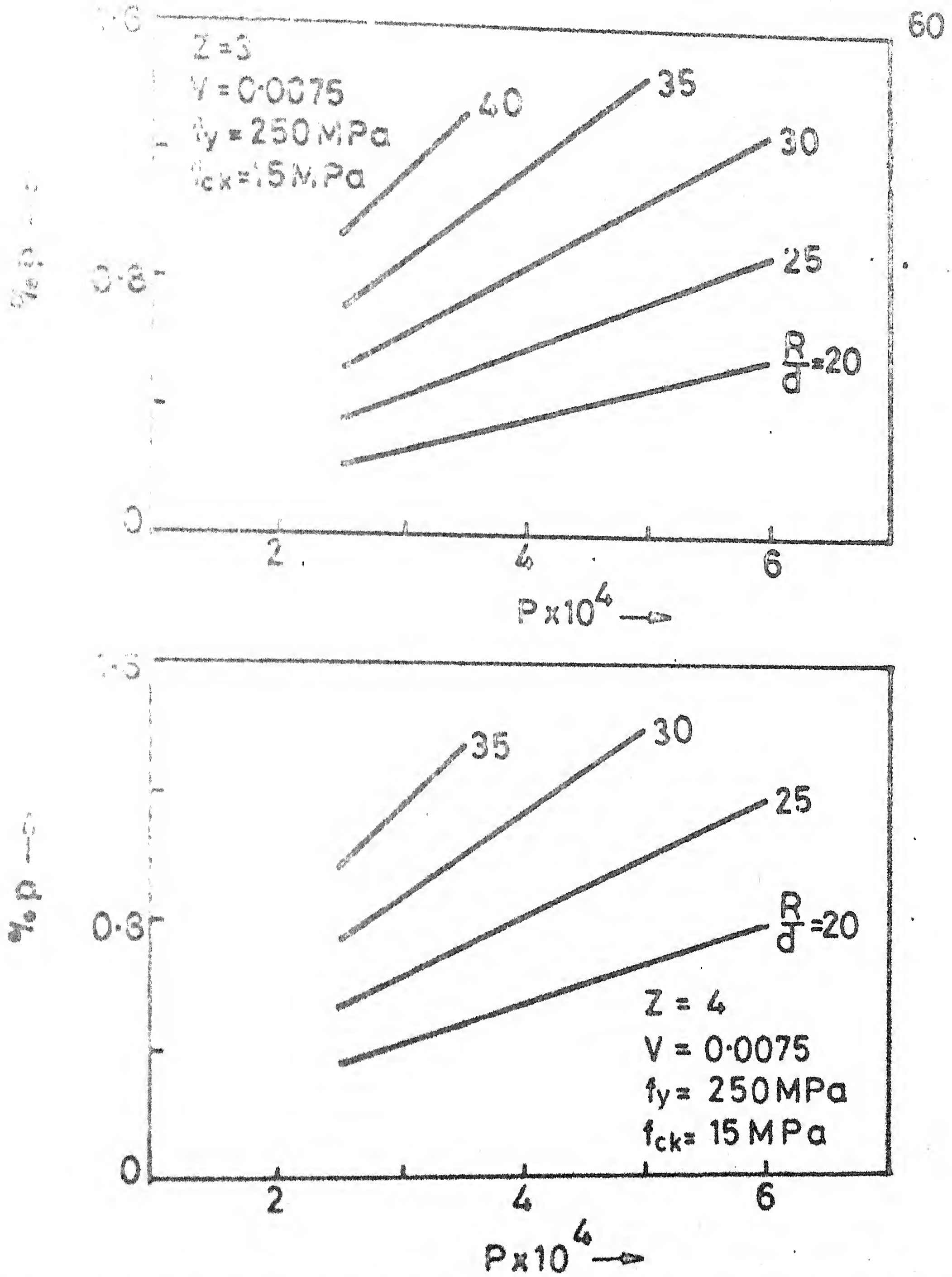
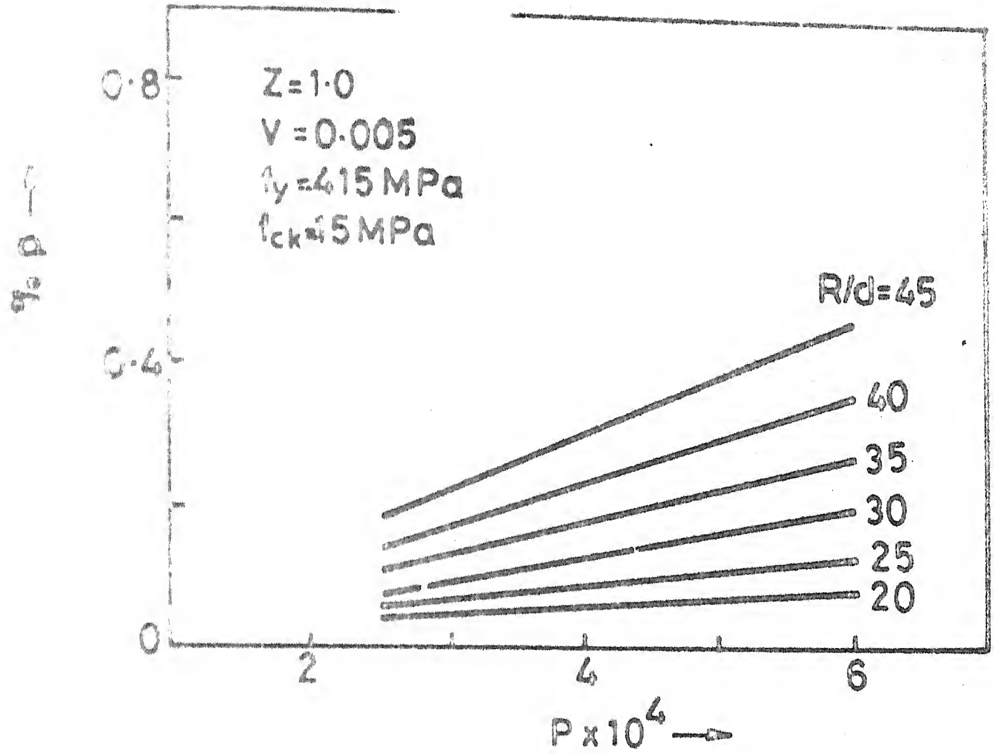


FIG.3·13 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF SIMPLY SUPPORTED UNDER-REINFORCED CONCRETE CIRCULAR SLABS



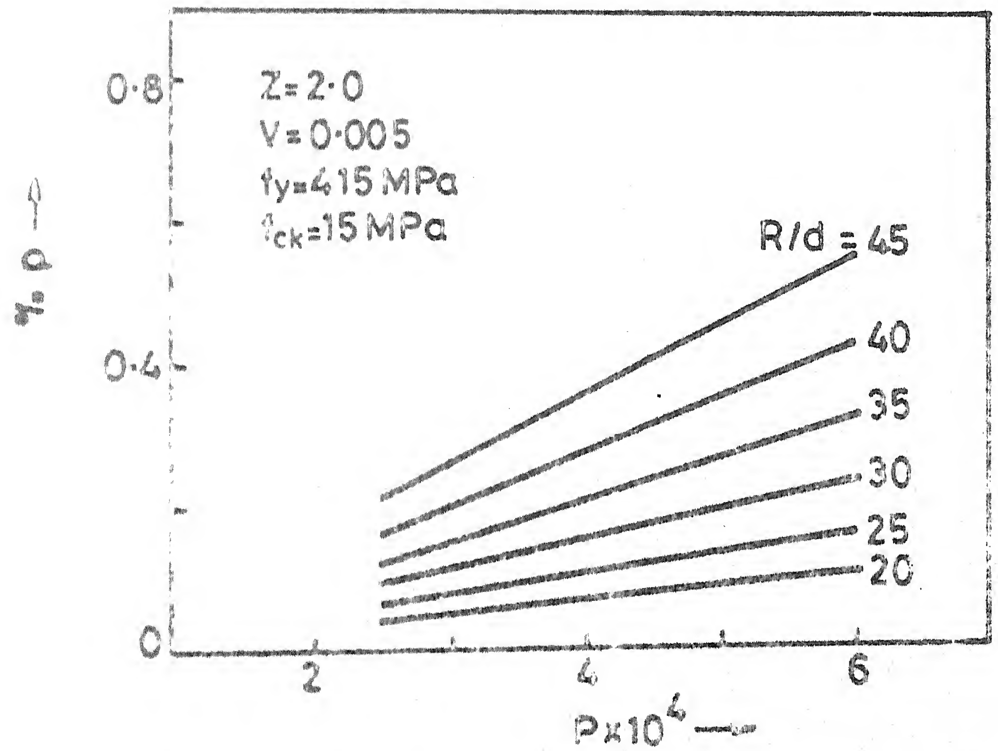


FIG.3·14 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF UNDER-REINFORCED CONCRETE CIRCULAR SLABS (Fixed edge condition with $\gamma = 1{\cdot}0$)

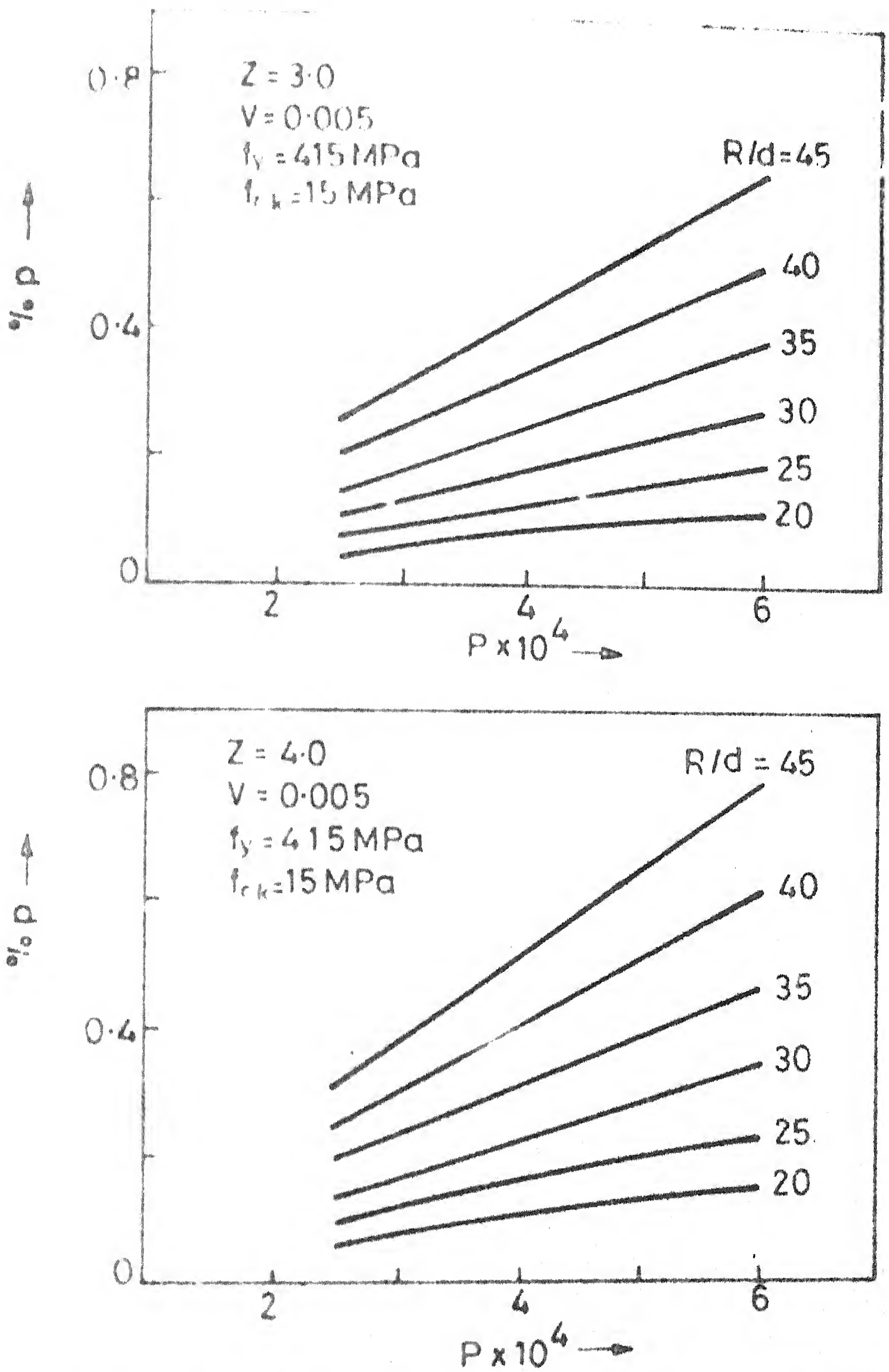


FIG 3.15 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF UNDER-REINFORCED CONCRETE CIRCULAR SLABS (Fixed edge condition with γ = 1·0)

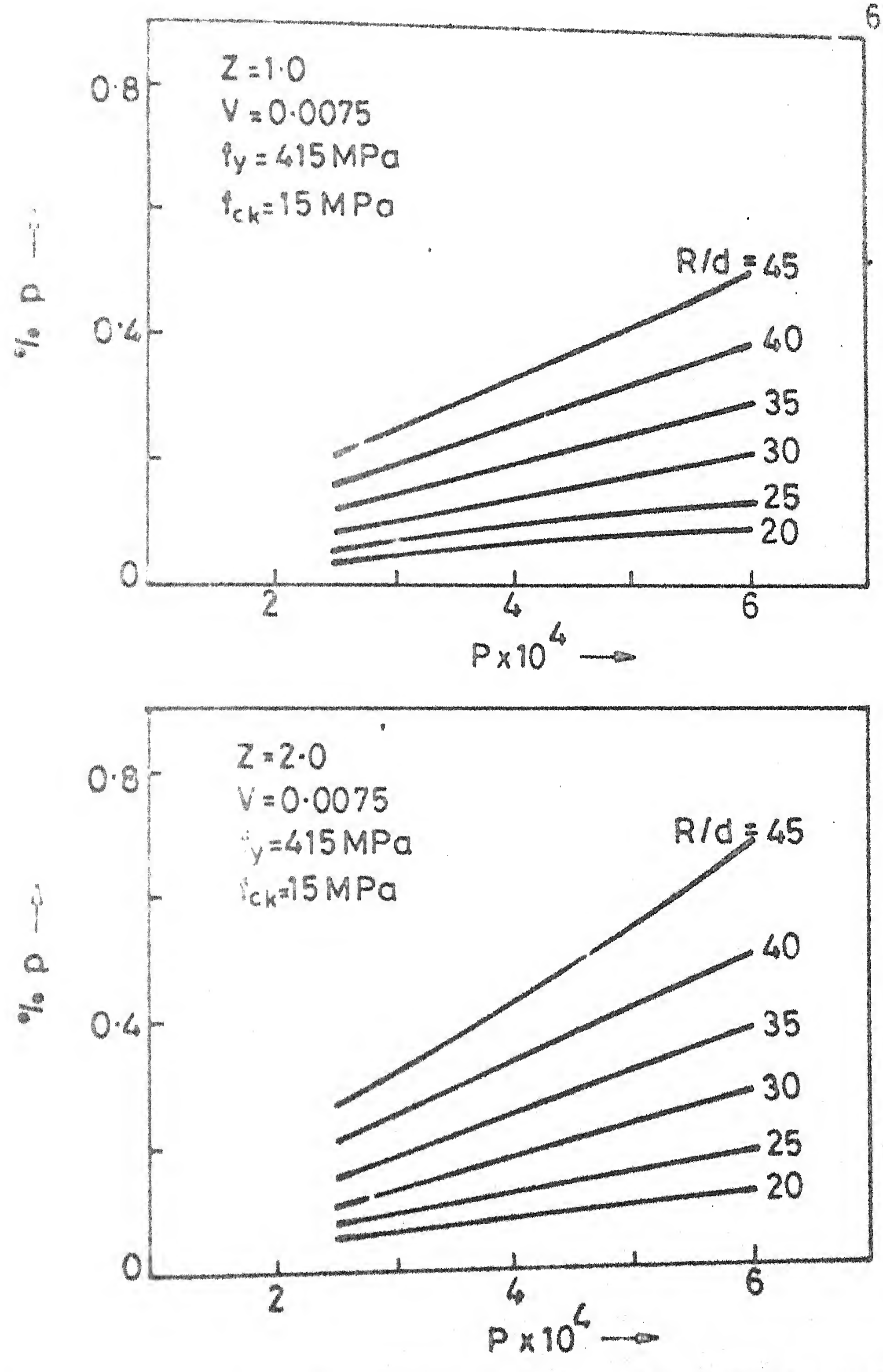


FIG.3.16 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF UNDER-REINFORCED CONCRETE CIRCULAR SLABS (Fixed edge condition with ν = 1·0)

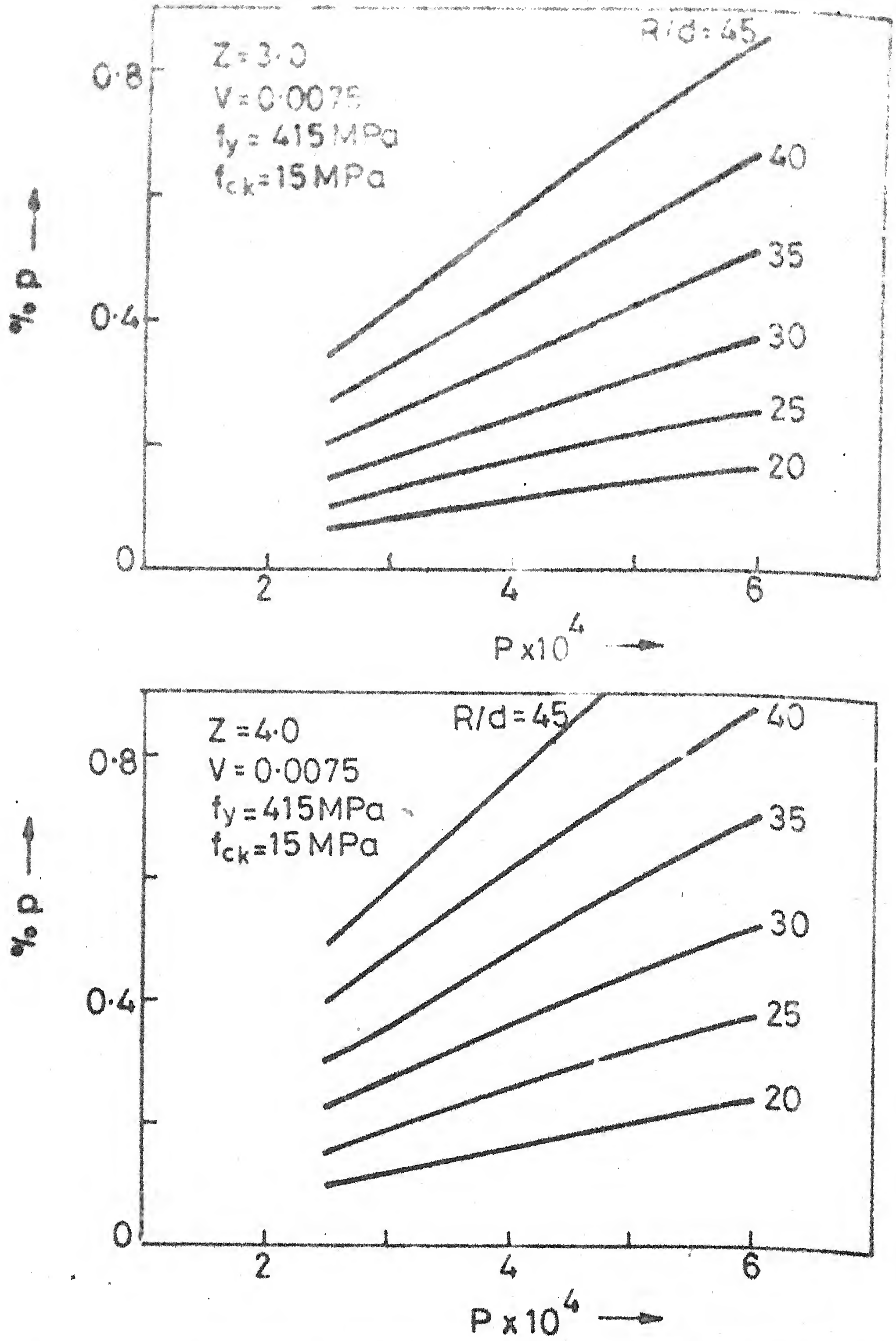


FIG. 3·17 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF UNDER-REINFORCED CONCRETE CIRCULAR SLABS (Fixed edge condition with ν = 1·0)

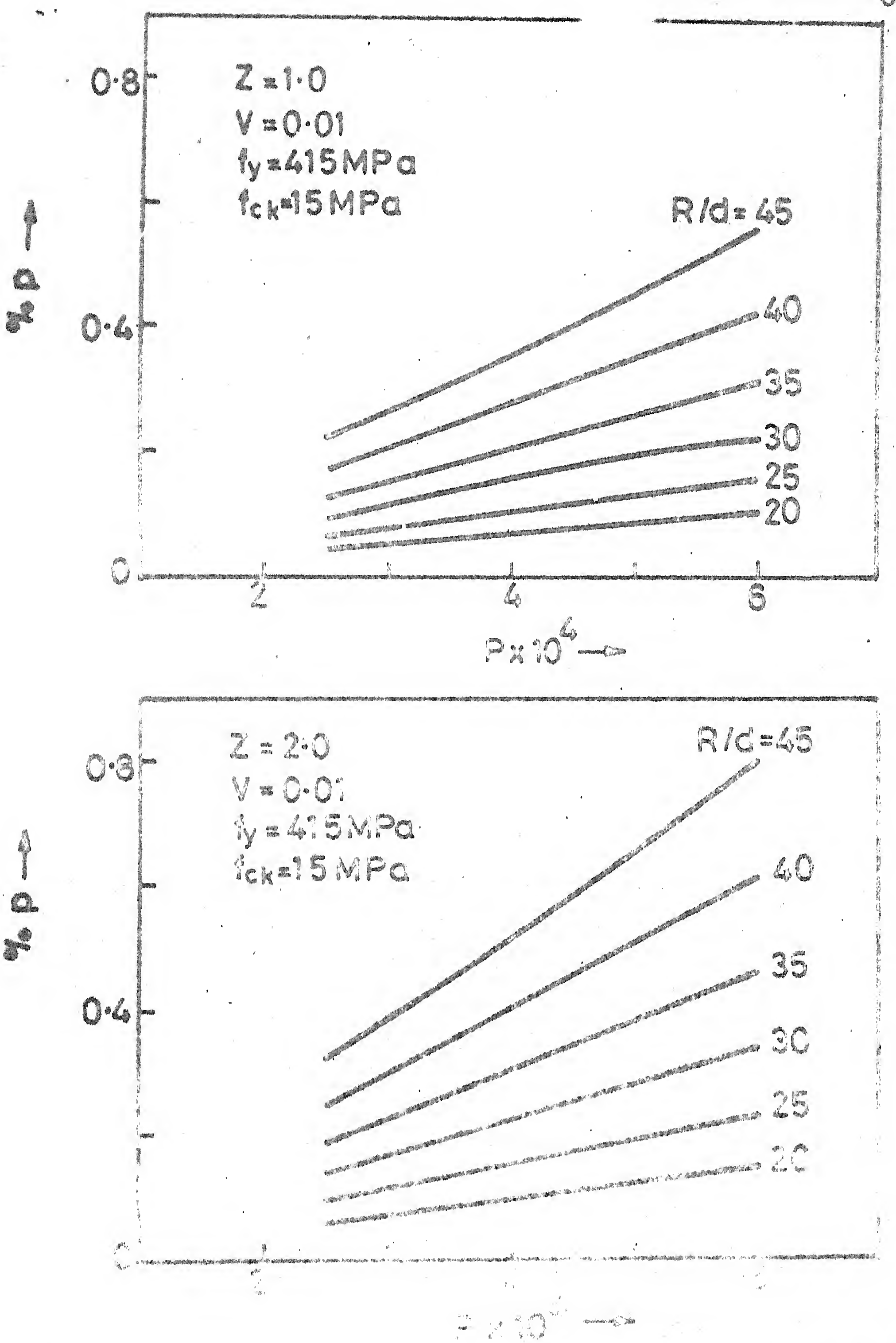


FIG. 3·18 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF UNDER-REINFORCED CONCRETE CIRCULAR SLABS (Fixed edge condition with $\gamma = 1\cdot0$)

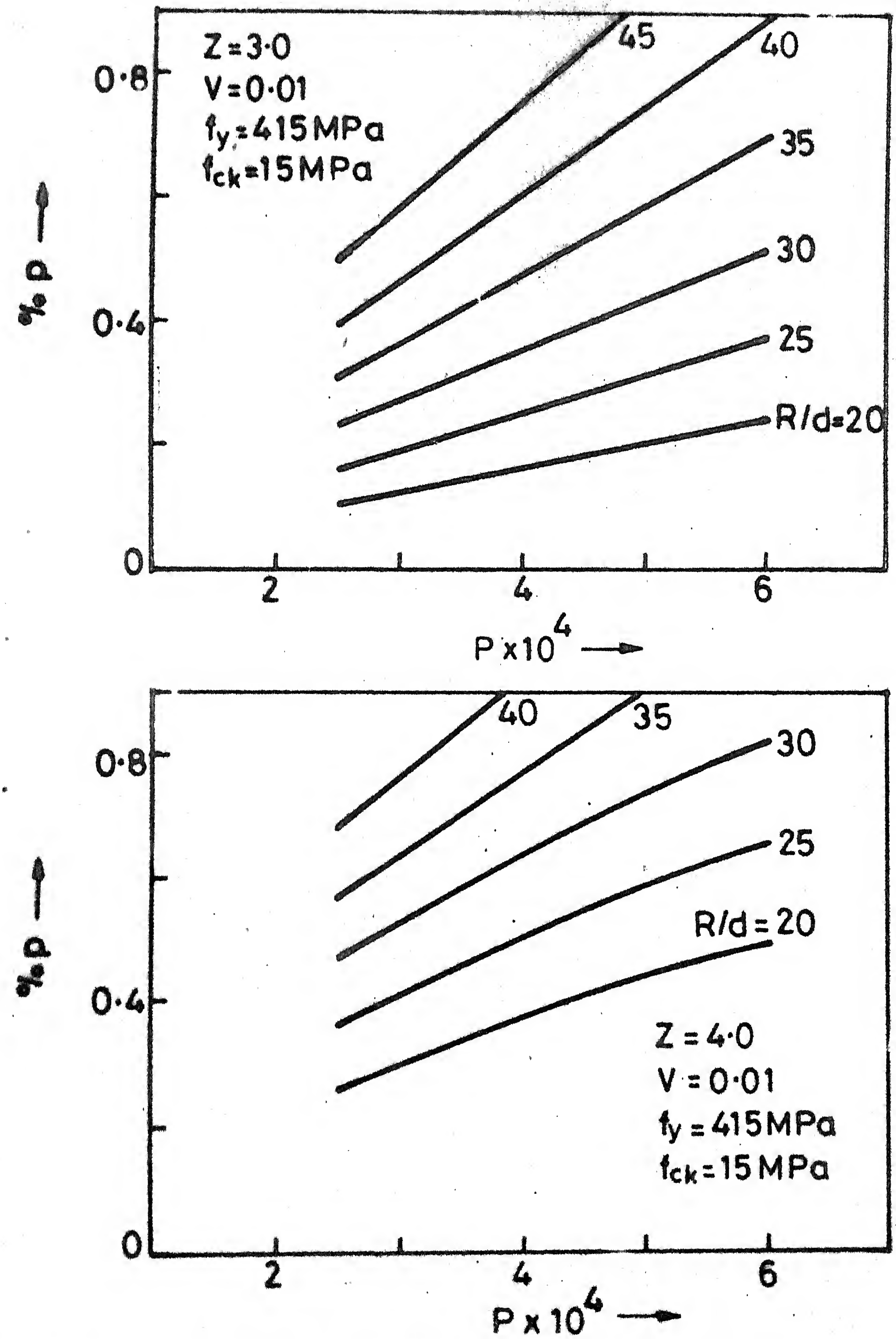


FIG.3·19 DESIGN CHARTS FOR RELIABILITY BASED DESIGN OF UNDER-REINFORCED CONCRETE CIRCULAR SLABS (Fixed edge condition ν=1·0)

CHAPTER 4

SOME ECONOMIC IMPLICATIONS IN REINFORCED CONCRETE CIRCULAR SLAB DESIGN

4.1 Introduction:

The basic aim of an optimum structural design is to arrive at a solution which satisfies a specified objective in the best possible manner. Usually, the specified objective functions are the minimum weight, minimum cost, minimum volume, maximum stiffness etc. The present study deals with the minimum cost design of concrete circular slabs with uncurtailed reinforcement for a specified reliability or probability of failure.

It is seen from the Chapter 3 that a more rational criterion for structural safety is the reliability. Most of the earlier minimum cost designs consider the design parameters as deterministic quantities. Recently however some studies have been performed considering the statistical nature of the material strengths.

In probability based design methods, usually a design is considered to be safe and adequate if the probability of failure of the structural member is less than a specified small quantity, say 10^{-6}. Since the quotation of such small probabilities may have little

significance, some alternative proposal for strength specification of structural components have also been made.

It has been suggested that the design strength be set equal to the expected value of the strength minus a constant number of standard deviations of the strength. Such a procedure is incorporated in the present study. The minimum cost design of RCC circular slabs is studied in some detail. The depth of the slab and the area of reinforcing steel are taken as the design variables. The variability in the material strengths, load and radius of the slab are considered in the form of coefficients of variation. It is to be noted that the approach can also be adopted for the optimum design of concrete beams with some modifications.

The design problem is formulated as a nonlinear mathematical programming problem. This formulation requires the determination of the mean and variance of the moment of resistance of the slab section for one metre width. The constrained optimization problem is solved by sequential unconstrained minimization technique (SUMT). Numerical results are presented by varying all the parameters affecting the design problem.

4.2 Formulation of the Optimization Problem:

The design of a circular slab is defined by two variables, namely steel reinforcement A_{st} and the slab depth d, provided the concrete and the steel strengths are specified. Concrete code requirements impose limits on the reinforcement and slab depth. For the range of concrete and steel strengths available and with in the limits imposed on design parameters there are still a very large number of acceptable solutions to the design problem The object, then is to find that solution, which results in a minimum cost and still satisfies all strength and safety requirements.

A classical mathematical programming problem is one in which a multivariate function $f(\vec{X})$ (where $\vec{X}$ is a n-dimensional vector consisting of X_i, i = 1, 2,...n) is to be minimized subject to given constraints $g_j(\vec{X}) \geq 0$, j = 1,2, ... m. The function $f(\vec{X})$ is called the objectiv function and its choice is governed by the nature of the problem. In the present problem, this function is taken to represent the cost of the RCC circular slab per metre length.

Taking depth of slab and steel reinforcement as the design variables X_i, (i = 1, 2) in this problem the objective function $f(\vec{X})$ can be expressed as

$$f(\vec{X}) = C_c.b.d + C_s.\ A_{st}(1+\nu) + C_f.d \quad \ldots \quad (4.2.1)$$

where C_c = cost of ready-mix concrete per unit volume

C_s = cost of reinforcing steel per unit weight

C_f = cost of thickness related form work per unit area

b = metre width of the slab

d = depth of the slab

A_{st} = area of positive reinforcement and

ν = ratio of negative to positive reinforcement.

As per the reliability considerations, the slab will be designed for the expected moment capacity M_R greater than or equal to the specified moment M_L plus a constant number of standard deviations of the moment capacity S_{M_R}. Thus the reliability based design criterion will be

$$M_R \geq M_L + C\ .\ S_{M_R} \quad \ldots \quad (4.2.2)$$

where C is a constant.

Now the optimization problem can be formulated as;

Find $\vec{X}$ for a minimum value of $f(\vec{X})$ such that

$$M_R - M_L - C\ S_{M_R} \geq 0 \quad \ldots \quad (4.2.3)$$

$$X_j^{(u)} - X_j \geq 0 \qquad (j = 1,2) \quad \ldots \quad (4.2.4)$$

$$X_j - X_j^{(l)} \geq 0 \qquad (j = 1,2) \quad \ldots \quad (4.2.5)$$

The upper bound on the area of steel $X_2^{(u)}$ is taken as the balanced steel area.

$$X_2^{(u)} = W\,d \qquad \ldots \qquad (4.2.6)$$

where

$$W = \left(0.5423 - \frac{0.0035}{\left(0.0055 + \frac{f_y}{E_s}\right)} \times \frac{f_{ck}}{f_y}\right) b \qquad \ldots \qquad (4.2.7)$$

The lower bound on steel area is taken as the minimum area of steel required according to IS Code of Practice(32)

$$X_2^{(1)} = \frac{0.12\ bd}{100} \qquad \ldots \qquad (4.2.8)$$

Lower limit on depth is taken as the minimum depth required to control the deflection according to IS Code(32).

$$X_1^{(1)} = \frac{2R}{30} \qquad \ldots \qquad (4.2.9)$$

Equation (4.2.3) needs expressions for M_R, M_L, S_{M_R}.

Moment of resistance (Appendix B)

$$M_R = A_{st} \cdot f_y \left(d - \frac{K\ A_{st}\ f_y}{f_{ck}\ b}\right) \qquad \ldots \qquad (4.2.10)$$

where $K = 0.776$.

$$M_L = \frac{p^* R^2}{6(1+\nu)} \qquad \ldots \qquad (4.2.11)$$

By virtue of equation (3.2.6), equation for S_{M_R} will be

$$S_{M_R}^2 = \left(\frac{\partial M_R}{\partial A_{st}}\right)^2 S_{A_{st}}^2 + \left(\frac{\partial M_R}{\partial f_y}\right)^2 S_{f_y}^2 + \left(\frac{\partial M_R}{\partial f_{ck}}\right)^2 S_{f_{ck}}^2$$

$$+ \left(\frac{\partial M_R}{\partial d}\right)^2 S_d^2 + \left(\frac{\partial M_R}{\partial b}\right)^2 S_b^2 \qquad \ldots \qquad (4.2.12)$$

$$S_{M_R} = \left[\left(f_y d - \frac{2K\ A_{st}\ f_y^2}{f_{ck}\ b}\right)^2 A_{st}^2\ V_{A_{st}}^2 \right.$$

$$+ \left(A_{st} \cdot d - \frac{2K\ A_{st}^2\ f_y}{f_{ck} \cdot b}\right)^2 f_y\ V_{f_y}^2$$

$$+ \left(\frac{K\ A_{st}^2\ f_y^2}{f_{ck}^2 \cdot b}\right)^2 f_{ck}^2\ V_{f_{ck}}^2$$

$$\left. + f_y^2\ A_{st}^2\ d^2\ V_d^2 + \left(\frac{K\ A_{st}^2\ f_y^2}{f_{ck} \cdot b^2}\right)^2 b^2\ V_b^2 \right]^{1/2} \quad \ldots \quad (4.2.13)$$

where $V_{A_{st}}$, V_{f_y}, $V_{f_{ck}}$, V_d, V_b are the coefficients of variation of area of steel, steel tensile strength, compressible strength of concrete, depth of slab and 1 metre width of the slab respectively.

If the coefficients of variation $V_{A_{st}} = V_{f_y} = V_{fck} = V_d$ $= V_b = V$ i.e. same for all the variables, then,

$$S_{M_R} = V \left[2A_{st}^2\ \left(f_y\ d - \frac{2K\ A_{st}\ f_y^2}{f_{ck} \cdot b}\right)^2 \right.$$

$$\left. + \frac{2K^2\ A_{st}^4\ f_y^4}{f_{ck}^2\ b^2} + f_y^2\ A_{st}^2\ d^2 \right]^{1/2} \quad \ldots \quad (4.2.14)$$

Finally the constraints can be stated after normalization as follows:

$$\left(\frac{M_L + C\ SM_R}{M_R}\right) - 1 \leq 0 \quad \ldots \quad (4.2.15)$$

$$\frac{X_2}{W\ X_1} - 1 \leq 0 \quad \ldots \quad (4.2.16)$$

$$\frac{R}{15\ X_1} - 1 \leq 0 \qquad \ldots \qquad (4.2.17)$$

$$\frac{1.2\ X_1}{X_2} - 1 \leq 0 \qquad \ldots \qquad (4.2.18)$$

$$\frac{X_1}{300} - 1 \leq 0 \qquad \ldots \qquad (4.2.19)$$

4.3 Method of Solution of the Optimization Problem:

The optimization problem formulated in the foregoing section is solved by the sequential unconstrained minimization technique. The constrained problem is transformed to an unconstrained minimization problem by appending the constraints to the objective function through a penalty parameter r_k. Thus a new objective function is defined as

$$\phi\ (\vec{X}\ ,\ r_k) = f(\vec{X}) - r_k \sum_{j=1}^{m} \frac{1}{g_j(X)} \qquad \ldots \qquad (4.3.1)$$

(j = 1,2,...number of constraints).

where $\phi\ (\vec{X},r_k)$ is called as penalty function. The second term of the equation (4.3.1) is the penalty term and becomes large as the vector of design variables $\vec{X}$ approaches a constraint. The vector of design variables corresponding to the minimum value of the objective function is found by carrying out a sequence of minimizations of the penalty function for a decreasing sequence of values $r_k (r_{k+1} < r_k)$.

For any fixed r_{k_1} the unconstrained minimum of the penalty function is found by Davidon-Fletcher - Powell method [27]. The derivatives are calculated by finite differences method. The initial feasible design points for the optimization procedure are found by a process of trial and error.

4.4 Results:

Several examples are taken in order to illustrate the effectiveness of the method described. In these examples, the cost coefficients are taken as follows:

C_f = Rs. $60/m^2$ and for 200 mm depth

C_s = Rs.600/kN for f_y = 250 MPa

C_c = Rs.$550/m^3$ for f_{ck}= 15 MPa.

Reference Example:

Results obtained are as shown in Table IV by taking the following initial values.

$p^* = 12\ kN/m^2$; Radius = 3 m

Number of standard deviations C = 3

V = 0.05 ; d = 230 mm

A_{st} = 900 mm^2 ; ν = 0.0 (i.e. simply supported)

f_y = 250 MPa ; f_{ck}= 15 MPa.

Upper bound on depth is taken as 300 mm.
For this example the cost of the slab is reduced from Rs. 278.66 to Rs. 231.04 per metre length by the optimization procedure. The optimum vector found is as $d = 213.54$ mm; $A_{st} = 537.17$ mm^2. A_{st} should be provided in both the directions of the slab.

The variation in the optimum design with respect to the reference design is studied by changing the various design parameters and the results are tabulated in Table IV.

(i) Effect of Number of Standard Deviations 'C' :

The effect of increasing reliabilities are shown in Table V. As the reliability is increased by increasing the number of standard deviations 'C', the expected strength of the slab has to increase and hence the corresponding increase in the cost. This is evidenced by the results of Table V.

(ii) Effect of cost coefficients on the optimum design:

With variation of individual costs of forming, steel and concrete the effect on the optimum design is studied. The results are shown in Table VI. It is observed that the cost of forming has the dominant effect on the optimum cost design. The change in cost of steel has the least effect on the optimum design cost out of these three costs.

(iii) Effect of coefficient of variation:

The effect of coefficient of variation is also studied on the optimum design. It is observed that the cost goes up with the increase in the coefficient of variation value. The results are tabulated in Table VII.

(iv) Effect of building code requirements:

When the limitation on the depth of the slab to control the deflections is incorporated as a constraint in the minimization problem, it becomes an active constraint forcing d to a value larger than that required for strength alone and increasing the cost by as much as 30 percent.

The limitation on the steel reinforcement is always satisfied at optimum and has no effect on the cost , i.e. it becomes an inactive constraint.

TABLE IV

MINIMUM COST OF THE REFERENCE DESIGN

Starting values: Load $p^* = 12.00\ kN/m^2$, Radius $R = 3m$

Reliability parameter $C = 3.0$,

$V_i\ (i=1,2...5) = 0.05$, $f_y = 250$ MPa

$f_{ck} = 15$ MPa, $X_1^{(u)} = 300$ mm

Starting design vector : $X_1 = 230$ mm ; $X_2 = 900\ mm^2$

Objective function corresponding to the starting design vector = 278.66

Number of Iterations	Penalty Parameter	X_1 (mm)	X_2 (mm^2)	Penalty Function $\phi(X)$	Objective Function Cost (Rs.)
1	10.90630	222.4044	899.8530	405.5618	272.1902
2	5.45315	221.5650	830.8840	332.7741	265.1040
3	2.72657	220.2190	600.3594	275.8269	240.9962
4	1.363288	217.1004	558.3658	257.3444	236.1283
5	0.68164	213.5607	537.1708	237.3433	231.2593
6	0.340822	213.5508	537.1706	232.7004	231.1527
7	0.170411	213.5401	537.1700	231.9265	231.0434
8	0.8520×10^{-1}	213.5401	537.1700	231.1605	231.0434
9	0.852×10^{-2}	213.5401	537.1700	231.1435	231.0434
11	0.852×10^{-5}	213.5401	537.1700	231.0434	231.0434
12	0.852×10^{-6}	213.5401	537.1700	231.0434	231.0434

TABLE V

EFFECT OF C ON OPTIMUM DESIGN

Reference values : $p^* = 12.00\ kN/m^2$, $R = 3\ m$

$V_i (i=1,2\ldots5) = 0.05$, $f_y = 250$ MPa

$f_{ck} = 15$ MPa , $X_1^{(u)} = 300$ mm

Starting design vector : $X_1 = 230$ mm , $X_2 = 900\ mm^2$

Starting value of objective function = 278.66.

Values of C	Objective Function Cost (Rs.)		Optimum $\vec{X}$		Percentage change ~~Decrease~~ in Objective Function Compared to Ref. Design
	Starting Value	Optimum Value	X_1 (mm)	X_2 (mm^2)	
1	278.66	208.5196	201.73	400.88	25.17
2	278.66	222.0570	210.08	470.63	20.31
3	278.66	231.0434	213.54	537.17	17.08
4	278.66	238.6312	214.38	610.38	14.36
5	278.66	247.3270	214.69	701.674	11.244

TABLE VI

EFFECT OF COST COEFFICIENTS ON OPTIMUM DESIGN

All reference values and the starting design vector are same as those given in Table V, also C = 3.

Increase in Cost	Objective Function Optimum Value	Percentage Increase Compared to Reference Optimum Design	Optimum $\vec{X}$	
			X_1 (mm)	X_2 (mm^2)
Reference Design	231.0434	0.00	213.54	537.17
Concrete Cost + 50 Percent	271.7094	17.54	200.15	503.57
Steel cost + 50 Percent	240.6464	4.10	202.46	494.58
Forming Cost + 50 Percent	275.3184	19.10	219.61	602.86

TABLE VII

EFFECT OF COEFFICIENT OF VARIATION ON OPTIMUM COST

Reference design values are same as mentioned in Table V, except variation in V_i $(i=1,2,....5)$.

Coefficient of Variation	Objective Function Optimum Value Cost (Rs.)	Optimum $\vec{X}$	
		X_1 (mm)	X_2 (mm^2)
0.05	231.0434	213.54	537.17
0.10	258.4446	213.92	816.30
0.15	329.1570	211.37	1617.82

CHAPTER 5

RESULTS AND DISCUSSION

5.1 General:

The design charts presented in Chapter 3 are applicable to design the RCC circular slabs in flexure only. In presenting the charts it is assumed that the reinforcement is uncurtailed and isotropic. Both fixed and simply supported edge conditions are considered with uniformly distributed load. The limit state design of strength in shear is ignored, because in many cases shear stress is not critical in the design of RCC slabs. The following set of values for different design variables were used to evaluate the results:

V = 0.005, 0.0075, 0.01, 0.0125 and 0.015

Z = 1.0 , 2.0 ,3.0 , 4.0

f_y = 250 , 415 MPa

f_{ck} = 15 MPa

ν = 0.0 , 1.0

$P \times 10^3$ = 0.25, 0.30, 0.35, 0.40, 0.45, 0.5, 0.55 and 0.6

R/d = 20, 25, 30, 35, 40 and 45.

Charts are presented for different combinations of the design variables. Design charts only for few combinations of design variables f_y and f_{ck} are presented to illustrate the design procedure qualitatively.

Two design examples are presented for the illustration of the use of the design charts.

5.2 Design Examples:

Example : 1

Design of a circular slab, simply supported along the boundary with the following design variables and for different probabilities of failure given.

Radius of the slab $R = 3$ m

Live load $w_1 = 4$ kN/m^2

$f_{ck} = 15$ MPa

$f_y = 415$ MPa

$V = 0.005$

Probabilities of failure (i) $P_f = 0.158655$

(ii) $P_f = 0.022750$

(iii) $P_f = 0.001350$ and

(iv) $P_f = 0.000032$

For the given probabilities of failure, the corresponding Z values are obtained from Appendix A as Z = 1,2,3 and 4.

Assume the slab is isotropically reinforced at the bottom. Assume the effective depth of the slab d = 130 mm.

Overall depth $= 130 + 20 = 150$ mm

Self weight of the slab $w_d = 3.75$ kN/m^2

Total load on the slab $p^* = w_l + w_d$

$= 7.75$ kN/m^2

$$\frac{R}{d} = \frac{3000}{130} = 23.07$$

$$P = \left(\frac{p^*}{f_{ck}}\right) = 5.166\times10^{-4}$$

Now from Figs. 3.3 and 3.4 for $P = 5.166\times10^{-4}$ and $\frac{R}{d} = 23.07$

(i) $Z = 1$; $p = 0.20$; $A_{st} = 260$ mm^2

(ii) $Z = 2$; $p = 0.25$; $A_{st} = 325$ mm^2

(iii) $Z = 3$; $p = 0.31$; $A_{st} = 403$ mm^2

(iv) $Z = 4$; $p = 0.38$; $A_{st} = 494$ mm^2

Thus it is seen that the percentage of steel is increasing with decrease in probability of failure.

Example 2:

Design of a circular slab, fixed along the boundary with the following design variables and for different probabilities of failure given.

$R = 4$m ; $w_l = 4$ kN/m^2

$f_{ck} = 15$MPa; $f_y = 415$ MPa

$V = 0.01$

Probabilities of failure (i) $P_f = 0.158655$

(ii) $P_f = 0.022750$

(iii) $P_f = 0.001350$ and

(iv) $P_f = 0.000032$

for the given probabilities of failure, the corresponding Z values are obtained from Appendix A as Z = 1,2,3 and 4. Assume the slab is isotropically reinforced at the bottom as well as at the top near the edges. Let the moment capacity at the support is equal to that at the mid span.

Then $i = 1.0$

Let $d = 130$ mm

Total depth $= 130 + 20 = 150$ mm

$w_l = 4\ kN/m^2$; $w_d = 3.75\ kN/m^2$

Total weight on the slab $p^* = 7.76\ kN/m^2$

$$\frac{R}{d} = \frac{4000}{130} = 30.76$$

$$P = \left(\frac{p^*}{f_{ck}}\right) = 5.166 \times 10^{-4}$$

Now from Figs. 3.18 and 3.19 for $P = 5.166 \times 10^{-4}$ and $R/d = 30.76$.

(i)	Z = 1 ;	p = 0.21 ;	$A_{st} = 273\ mm^2$
(ii)	Z = 2 ;	p = 0.305 ;	$A_{st} = 396.5\ mm^2$
(iii)	Z = 3 ;	p = 0.465 ;	$A_{st} = 604.5\ mm^2$
(iv)	Z = 4 ;	p = 0.74 ;	$A_{st} = 962\ mm^2$

Provide the same percentage of reinforcement at the top of the slab up to a length of R/3 = 1.33 m from the boundary.

5.3 Conclusions and Recommendations:

The following conclusions and recommendations can be mentioned from the study of the reliability based design of RCC circular slabs.

(i) It is observed from the design formulation presented in Chapter 3 the area of reinforcement increased with the decrease in probability of failure and with increase in coefficients of variation.

(ii) At high coefficient of variation it is not possible to design the section for less reliability, we have to design the section for a minimum probability of failure.

(iii) It is recommended to decrease the coefficient of variation to get a low percentage of steel area even at low probabilities of failure.

(iv) Even with the rich mix concretes there is only a slight change in percentage of reinforcement. So it is better to use the grade of concrete depending upon the conditions of exposure (environmental conditions).

The feasibility of designing concrete slabs for minimum cost with a reliability based constraint has been demonstrated in Chapter 4. The problem of specifying small numbers of the order 10^{-6} for the probability of failure has been avoided by taking the statistical constraint as shown in equation (4.2.2). This constraint can be directly converted into a probability of failure constraint if the distribution functions of the strength parameters are known. For example, a value of $C = 2$ in equation (4.2.2) corresponds to a probability of failure of 0.0227, if the moment capacity M_R follows standard normal distribution.

On the basis of the results summarized in Chapters 3 and 4, it is possible to adopt some guide lines for the design of concrete circular slabs. It is believed that these guide lines will result in a design of minimum cost. Furthermore, the sensitivity analyses performed indicate that the results will remain valid for a wide range of changes in the cost of materials, in the light of changing economic conditions.

Design Guidelines:

(i) Choose a steel reinforcement of high strength.

(ii) Choose a concrete strength in the medium to low range.

(iii) The coefficient of variation should be as minimum as possible.

5.4 Scope for Further Work:

In the present work the curtailment of reinforcement is not considered, it should be considered in the reliability based design. The depth of the slab is assumed as uniform throughout, and only the isotropic reinforcement is considered. These two, orthotropic reinforcement and variable depth should also be considered in further studies. The strength design in shear is also not considered in this work. It can also be considered in further studies where in it is likely to be critical.

A more realistic method of design includes the control of mix, field conditions of curing, local product availability, etc. Further work can be done including these considerations. It is hoped that the design formulation presented will lead a way for many more sophisticated designs.

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APPENDIX A

TABLE OF STANDARD NORMAL PROBABILITIES

Z	f(Z)	Reliability R(Z)	Probability of failure 'P_f'
0.0	0.398942	0.500000	0.500000
0.1	0.396952	0.539827	0.460173
0.2	0.391043	0.579260	0.420740
0.3	0.381388	0.617911	0.382089
0.4	0.368270	0.655422	0.344578
0.5	0.352065	0.691465	0.308535
0.6	0.333225	0.725747	0.274253
0.7	0.312254	0.758036	0.241964
0.8	0.289692	0.788145	0.211855
0.9	0.266085	0.815940	0.184060
1.0	0.241971	0.841345	0.158655
1.1	0.217852	0.864334	0.135666
1.2	0.194186	0.884930	0.115070
1.3	0.171369	0.903195	0.096805
1.4	0.149727	0.919243	0.080757
1.5	0.129518	0.933193	0.066807
1.6	0.110921	0.945201	0.054799
1.7	0.094049	0.955435	0.044565
1.8	0.078950	0.964069	0.035931
1.9	0.065616	0.971284	0.028716
2.0	0.053991	0.977250	0.022750
2.2	0.035475	0.986097	0.013903
2.4	0.022395	0.991803	0.008197
2.6	0.013583	0.995339	0.004661
2.8	0.007915	0.997495	0.002505
3.0	0.004432	0.998650	0.001350
4.0	0.000134	0.999968	0.000032
5.0	0.0000015	0.9999997	0.0000003

APPENDIX B

MOMENTS OF RESISTANCE FOR A RECTANGULAR SECTION (With out Partial Safety Factors)

Without partial safety factor for characteristic strength of concrete the area of compressive stress[32] will be $0.54238\ f_{ck}\ X_u\ b$.

For equillibrium of force

$$\text{Tensile force} = \text{Compressive force}$$

$$A_{st}\ f_y = 0.5423\ f_{ck}\ X_u\ b$$

Here partial safety factor for f_y is also not considered

$$\frac{X_u}{d} = \frac{A_{st}\ f_y}{0.5423\ f_{ck}\ db}$$

$$\frac{X_u}{d} < K_b$$

K_b - limiting value

$$K_b = \frac{0.0035}{0.0055 + f_y/E_s}$$

For under-reinforced section

$$M_R = A_{st} \cdot f_y \left(d - \frac{0.42\ A_{st}\ f_y}{0.5423\ f_{ck} b} \right)$$

$$= A_{st}\ f_y d \left(1 - 0.774\ \frac{A_{st}\ f_y}{f_{ck}\ bd}\right)$$

If $\frac{X_u}{d} \geq K_b$

$$M_{R,\text{limit}} = 0.5423\ \frac{X_{u,max}}{d} \left(1 - 0.42\ \frac{X_{u,max}}{d}\right) bd^2\ f_{ck}\ .$$